9.3 The pressure distribution on the 1-m-diameter circular disk in Fig. P9.3 is given in the table. Determine the drag on the disk.

\[ D = \sum_{i} \rho \, dA - \int_{r=0}^{r=D/2} \rho \, (2 \pi r \, dr) - \rho_2 \, \frac{D^2}{4}, \text{ since } dA = 2\pi r \, dr \]

Thus, \( D = 2\pi \int_{0}^{0.5} \rho \, r \, dr - (-5 \, \text{kN/m}^2) \frac{D^2}{4} (1 \text{m}^2) = 2\pi \int_{0}^{0.5} \rho \, r \, dr + 3.93 \text{ kN} \)

Evaluate the integral numerically using the following integrand:

<table>
<thead>
<tr>
<th>( r, \text{ m} )</th>
<th>( \rho r, \text{ kN/m} )</th>
<th>( r, \text{ m} )</th>
<th>( p, \text{ kN/m}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4.34</td>
</tr>
<tr>
<td>0.05</td>
<td>0.214</td>
<td>0.05</td>
<td>4.28</td>
</tr>
<tr>
<td>0.10</td>
<td>0.406</td>
<td>0.10</td>
<td>4.06</td>
</tr>
<tr>
<td>0.15</td>
<td>0.558</td>
<td>0.15</td>
<td>3.72</td>
</tr>
<tr>
<td>0.20</td>
<td>0.620</td>
<td>0.20</td>
<td>3.10</td>
</tr>
<tr>
<td>0.25</td>
<td>0.695</td>
<td>0.25</td>
<td>2.78</td>
</tr>
<tr>
<td>0.30</td>
<td>0.711</td>
<td>0.30</td>
<td>2.37</td>
</tr>
<tr>
<td>0.35</td>
<td>0.662</td>
<td>0.35</td>
<td>1.89</td>
</tr>
<tr>
<td>0.40</td>
<td>0.594</td>
<td>0.40</td>
<td>1.41</td>
</tr>
<tr>
<td>0.45</td>
<td>0.333</td>
<td>0.45</td>
<td>0.74</td>
</tr>
<tr>
<td>0.50</td>
<td>0.080</td>
<td>0.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Using a standard numerical integration technique with the above integrand gives \( D = 5.43 \text{ kN} \)
9.4 A 0.10 m-diameter circular cylinder moves through air with a speed $U$. The pressure distribution on the cylinder’s surface is approximated by the three straight line segments shown in Fig. P9.4. Determine the drag coefficient on the cylinder. Neglect shear forces.

\[ d\rho = b r \rho \cos \theta \, d\theta = b r \int_0^\pi \rho \cos \theta \, d\theta \]

or

\[ d\rho = 2 b r \int_0^{\pi/2} \rho \cos \theta \, d\theta \]

Break up the integration into the following three segments:

1) $0 \leq \theta \leq 70^\circ = 1.222 \text{ rad}$ where
   \[ \rho = -7.39 \theta + 3 \frac{N}{m^2}, \text{ where } \theta \text{- rad.} \]
   \[ \text{i.e. } \rho \bigg|_{\theta = 0} = 3 \text{ and } \rho \bigg|_{\theta = 1.222} = -6 \]

2) $70^\circ \leq \theta \leq 100^\circ$ or $1.222 \leq \theta \leq 1.745 \text{ rad}$ where
   \[ \rho = 8.59 \theta - 16.5 \frac{N}{m^2}, \text{ where } \theta \text{- rad} \]
   \[ \text{i.e. } \rho \bigg|_{\theta = 1.222} = -6 \text{ and } \rho \bigg|_{\theta = 1.745} = -1.5 \]

and

3) $100^\circ \leq \theta \leq 180^\circ$ or $1.745 \leq \theta \leq 3.14 \text{ rad}$ where
   \[ \rho = -1.5 \frac{N}{m^2} \]

Thus,

\[ d\rho = 2 b r \left[ \int_0^{70^\circ} \rho \cos \theta \, d\theta + \int_{70^\circ}^{100^\circ} \rho \cos \theta \, d\theta + \int_{100^\circ}^{180^\circ} \rho \cos \theta \, d\theta \right] = 2 b r \left[ I_x + I_y + I_z \right] \]

where

(continues)
\[ I_1 = \int_0^{1.222} (-7.39 \cos \theta + 3 \cos \theta \sin \theta + 3 \sin \theta) \, d\theta = -0.791 \]
\[ I_2 = \int_0^{1.222} (8.59 \cos \theta - 16.5) \cos \theta \, d\theta = -0.260 \]
\[ I_3 = \int_0^{3.14} (-1.5) \cos \theta \, d\theta = 1.477 \]

Hence,
\[ \omega_p = 2br[0.791 - 0.260 + 1.477] = 0.852 \text{ br} \]

or with
\[ C_p = \frac{\omega_p}{\frac{1}{2} \rho U^2 A} = \frac{0.852 \text{ br}}{\frac{1}{2} \rho U^2 (2br)} = \frac{0.426}{\frac{1}{2} \rho U^2} \]

But the pressure at \( \theta = 0 \), the stagnation point, is \( 3N/m^2 \).

Thus, \( \frac{1}{2} \rho U^2 = 3 \frac{N}{m^2} \) so that

\[ C_p = \frac{0.426}{3} = 0.142 \]
9.5 Repeat Problem 9.1 if the object is a cone made by rotating the triangle \( A-B-C \) about the horizontal axis \( A-C \) rather than the wedge-shaped bar.

\[ \mathcal{D} = \frac{1}{2} \rho V^2 A C_B \quad \text{where} \quad C_B = 0.5 \quad \text{and} \quad A = \pi (0.3c)^2 = 0.09 \pi c^2 \quad (1) \]

Also,

\[ \mathcal{D} = -\rho_A A + \int_{0}^{x=0.3c/sin \theta} \rho_{BC} \sin \theta \, dA \quad \text{where} \quad dA = 2 \pi r \, dx = 2 \pi (x \sin \theta) \, dx \\
\text{Hence,} \quad \mathcal{D} = -(-0.2) \frac{1}{2} \rho U^2 (0.09 \pi c^2) + \int_{x=0}^{x=0.3c/sin \theta} \rho_{BC} \sin \theta \left[ 2 \pi (x \sin \theta) \, dx \right] \quad (2) \]

or with \( \rho_{BC} = \text{average pressure} = \text{constant} \), Eq. (2) becomes

\[ \mathcal{D} = (0.2) \frac{1}{2} \rho U^2 (0.09 \pi c^2) + \rho_{BC} \sin ^2 \theta (2\pi) \int_{x=0}^{x=0.3c/sin \theta} x \, dx \]

\[ = (0.2) \frac{1}{2} \rho U^2 (0.09 \pi c^2) + \rho_{BC} \sin ^2 \theta (2\pi) (0.3c/sin \theta)^2/2 \]

or

\[ \mathcal{D} = [(0.2) \frac{1}{2} \rho U^2 + \rho_{BC}] (0.09 \pi c^2) \quad (3) \]

By combining Eqs. (1) and (3) we obtain

\[ \frac{1}{2} \rho U^2 (0.09 \pi c^2) (0.5) = [(0.2) \frac{1}{2} \rho U^2 + \rho_{BC}] (0.09 \pi c^2) \]

which gives

\[ \rho_{BC} = (0.3) \frac{1}{2} \rho U^2 \]
9.1 The wedge-shaped bar shown in Fig. P9.1 moves through a fluid with its top (surface A → C) parallel to the free stream. The drag coefficient is 0.5 and the pressure on the base (surface A → B) is less than the free stream pressure by an amount equal to 20% of the dynamic pressure. Viscous effects are negligible. Determine the average pressure on surface B → C.

\[ \mathcal{D} = \frac{1}{2} \rho U^2 A C_D \quad \text{where} \quad A = A_{AB} \text{ and } C_D = 0.5 \]

Also, \( \mathcal{D} = -\rho_{AB} A_{AB} + \rho_{BC} A_{BC} \sin \theta \), where \( \rho_{AB} = (-0.2) \frac{1}{2} \rho U^2 \) and \( A_{BC} = A_{AB} / \sin \theta \)

Hence,

\[ -(-0.2) \frac{1}{2} \rho U^2 A_{AB} + \rho_{BC} A_{AB} \sin \theta = \frac{1}{2} \rho U^2 A_{AB} \quad (0.5) \]

or

\[ (0.2) \frac{1}{2} \rho U^2 + \rho_{BC} = (0.5) \frac{1}{2} \rho U^2 \quad \text{or} \quad \rho_{BC} = (0.3) \frac{1}{2} \rho U^2 \]
9.2 The average pressure and shear stress acting on the surface of the 1-m-square flat plate are as indicated in Fig. P9.2. Determine the lift and drag generated. Determine the lift and drag if the shear stress is neglected. Compare these two sets of results.

Since \( \sum \rho \, dA = \rho_{ave} A \) and \( \sum \tau_{w} \, dA = \tau_{ave} A \) it follows that

\[
\sum F = -\rho_{ave} A \sin \alpha + \rho_{ave} A_{2} \sin \alpha + \tau_{ave} A_{1} \cos \alpha + \tau_{ave} A_{2} \cos \alpha
\]
or with \( A_{1} = A_{2} = 1 \text{ m}^2 \) and \( \alpha = 7^\circ \),

\[
\sum F = A_{1} \sin \alpha (\rho_{2} - \rho_{1}) + A_{1} \cos \alpha (\tau_{2} - \tau_{1})
\]
\[
= (1 \text{ m}^2) \sin 7^\circ (2.3 - (-1.2)) \frac{\text{kN}}{\text{m}^2} + (1 \text{ m}^2) \cos 7^\circ (5.8 \times 10^{-2} + 7.6 \times 10^{-2}) \frac{\text{kN}}{\text{m}^2}
\]
\[
= 0.427 \text{ kN} + 0.133 \text{ kN} = 0.560 \text{ kN}
\]

Note: if shear stress is neglected \( \sum F = 0.427 \text{ kN} \) (ie., \( \tau_{1} = \tau_{2} = 0 \))

Also, \( \sum L = -\rho_{ave} A_{1} \cos \alpha + \rho_{ave} A_{2} \cos \alpha - \tau_{ave} A_{1} \sin \alpha - \tau_{ave} A_{2} \sin \alpha \)

or \( \sum L = A_{1} \cos \alpha (\rho_{2} - \rho_{1}) - A_{1} \sin \alpha (\tau_{2} - \tau_{1}) \)
\[
= (1 \text{ m}^2) \cos 7^\circ (2.3 - (-1.2)) \frac{\text{kN}}{\text{m}^2} - (1 \text{ m}^2) \sin 7^\circ (5.8 \times 10^{-2} + 7.6 \times 10^{-2}) \frac{\text{kN}}{\text{m}^2}
\]
\[
= 3.47 \text{ kN} - 0.0163 \text{ kN} = 3.45 \text{ kN}
\]

Note: if shear stress is neglected \( \sum L = 3.47 \text{ kN} \)

Note: If the general expressions \( \sum F = \sum \rho \cos \theta \, dA + \sum \tau_{w} \sin \theta \, dA \)
and \( \sum L = -\sum \rho \sin \theta \, dA + \sum \tau_{w} \cos \theta \, dA \) are use, be careful about the signs involved. On the upper surface \( \Theta_{1} = 97^\circ \) and \( \rho \) and \( \tau_{w} \) are positive as indicated in the figure. On the lower surface \( \Theta_{2} = 277^\circ \)
and \( \rho \) and \( \tau_{w} \) are positive as indicated in the lower figure.

For example, with this notation \( \tau_{w} < 0 \) on the lower surface.
\[
\sum F = -(-1.2 \text{ kN}) \sin 97^\circ (1 \text{ m}^2) - (2.3 \text{ kN}) \sin 277^\circ (1 \text{ m}^2)
\]
\[
+ (5.8 \times 10^{-2} \text{ kN}) \cos 97^\circ (1 \text{ m}^2) + (7.6 \times 10^{-2} \text{ kN}) \cos 277^\circ (1 \text{ m}^2)
\]
\[
= 3.45 \text{ kN} \) as obtained above.
9.6 A 5-ft-long porpoise swims with a speed of 30 ft/s. Would a boundary layer type flow be developed along the sides of the animal? Explain.

\[ \text{Re} = \frac{UL}{\nu}, \text{ or with } L = 5 \text{ ft}, U = 30 \text{ ft/s}, \text{ and } \nu = 1.21 \times 10^{-5} \text{ ft}^2/\text{s} \]

\[ \text{Re} = \frac{(5 \text{ ft})(30 \text{ ft/s})}{1.21 \times 10^{-5} \text{ ft}^2/\text{s}} = 1.24 \times 10^7 \]

This Reynolds number is large enough to ensure boundary layer flow. (\( \text{Re} \approx 1000 \) is often assumed to be the lower limit.)
9.7 Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar; turbulent? Explain.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Speed</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) large whale</td>
<td>10 m/s</td>
<td>300,000,000</td>
</tr>
<tr>
<td>(b) flying duck</td>
<td>20 m/s</td>
<td>300,000</td>
</tr>
<tr>
<td>(c) large dragonfly</td>
<td>7 m/s</td>
<td>30,000</td>
</tr>
<tr>
<td>(d) invertebrate larva</td>
<td>^ 1 mm/s^</td>
<td>0.3</td>
</tr>
<tr>
<td>(e) bacterium</td>
<td>0.01 mm/s</td>
<td>0.00003</td>
</tr>
</tbody>
</table>

Inertia important if \(Re \gg 1\) (i.e. whale, duck, dragonfly)
Viscous effects dominate if \(Re \ll 1\) (i.e. larva, bacterium)
Boundary layer flow becomes turbulent for \(Re\) on the order of \(10^5\) to \(10^6\) (i.e. whale and perhaps the duck)
The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.
9.9 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

\[ \text{Re} = \frac{UD}{\nu} < 1 \text{ or } U < \frac{\nu}{D} \text{ if viscous effects are to be important throughout the flow.} \]

For standard air, \( \nu = 1.57 \times 10^{-6} \text{ ft}^2 \text{s}^{-1} \)

Thus,

\[ U < \frac{1.57 \times 10^{-6}}{D} \]

where \( D \) is the diameter in feet.

<table>
<thead>
<tr>
<th>Object</th>
<th>( D, \text{ ft} )</th>
<th>( \frac{U, \text{ ft}}{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>twig</td>
<td>( 2.08 \times 10^{-2} )</td>
<td>( 7.54 \times 10^{-3} )</td>
</tr>
<tr>
<td>hair</td>
<td>( 3.33 \times 10^{-4} )</td>
<td>0.471</td>
</tr>
<tr>
<td>smokestack</td>
<td>6</td>
<td>( 2.62 \times 10^{-5} )</td>
</tr>
</tbody>
</table>
9.10 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = CV^x$, where $C$ is a constant. Thus,

$$C = \frac{\delta}{Vx} = \frac{12 \times 10^{-3} m}{\sqrt{1.3 m}} = 0.0105$$

or $\delta = 0.0105 \sqrt{x}$ where $x = m, \delta = mm$

<table>
<thead>
<tr>
<th>$X, m$</th>
<th>$\delta, m$</th>
<th>$\delta, mm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.00470</td>
<td>4.70</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0148</td>
<td>14.8</td>
</tr>
<tr>
<td>20.0</td>
<td>0.0470</td>
<td>47.0</td>
</tr>
</tbody>
</table>

9.11 If the upstream velocity of the flow in Problem 9.10 is $U = 1.5 m/s$, determine the kinematic viscosity of the fluid.

For laminar flow $\delta = 5 \sqrt{\frac{Vx}{U}}$, or $\nu = \frac{UD^2}{25x}$

Thus,

$$\nu = \frac{(1.5 \frac{m}{s})(12 \times 10^{-3} m)^2}{25 (1.3 m)} = 6.65 \times 10^{-6} \frac{m^2}{s}$$
9.12 Water flows past a flat plate with an upstream velocity of \( U = 0.02 \text{ m/s} \). Determine the water velocity at distances of \( 10 \text{ mm} \) from the plate at distances of \( x = 1.5 \text{ m} \) and \( x = 15 \text{ m} \) from the leading edge.

From the Blasius solution for boundary layer flow on a flat plate, \( u = U f(\eta) \), where \( \eta \), the similarity variable, is

\[
\eta = \sqrt[3]{\frac{V}{V_x}}.
\]

Values of \( f(\eta) \) are given in Table 9.1.

Since \( \text{Re}_x = \frac{V_x}{V} = \frac{(0.02 \text{ m/s})(15 \text{ m})}{1.12 \times 10^6 \text{ m}^3/\text{s}} = 2.68 \times 10^5 \) is less than the critical \( \text{Re}_{x_c} = 5 \times 10^5 \), it follows that the boundary layer flow is laminar.

At \( x_1 = 1.5 \text{ m} \) and \( y = 10 \times 10^{-3} \text{ m} \) we obtain:

\[
\eta_1 = (10 \times 10^{-3} \text{ m}) \sqrt[3]{\frac{0.02}{(1.12 \times 10^6 \text{ m}^3/\text{s})(1.5 \text{ m})}} = 1.091
\]

Linear interpolation from Table 9.1 gives:

\[
f' = 0.2647 + \frac{(0.3378 - 0.2647)}{(1.0 - 0.8)} (1.091 - 0.8) = 0.359
\]

Hence,

\[
u_1 = U f'(\eta_1) = (0.02 \text{ m/s})(0.359) = 0.00718 \text{ m/s}
\]

Similarly, at \( x_2 = 15 \text{ m} \) and \( y = 10 \times 10^{-3} \text{ m} \) we obtain:

\[
\eta_2 = (10 \times 10^{-3} \text{ m}) \sqrt[3]{\frac{0.02}{(1.12 \times 10^6 \text{ m}^3/\text{s})(15 \text{ m})}} = 0.345
\]

Linear interpolation from Table 9.1 gives:

\[
f' = 0.0 + \frac{(0.1328 - 0.0)}{(0.8 - 0.4)} (0.345 - 0.0) = 0.1145
\]

Hence,

\[
u_2 = U f'(\eta_2) = (0.02 \text{ m/s})(0.1145) = 0.00229 \text{ m/s}
\]
Also, the momentum thickness, $\Theta$, is

$$\Theta = \int_0^{\infty} \frac{U}{U^*} (1 - \frac{U}{U^*}) dy = 4.94 \int_0^{\infty} \sqrt{h} \left(1 - 4.94 \sqrt{h} \right) dy.$$

Numerical integration of the tabulated data gives $\Theta = 2.23 \times 10^{-3} m$

<table>
<thead>
<tr>
<th>$y, m$</th>
<th>$(1 - 4.94 \sqrt{h})$</th>
<th>$\sqrt{h} \left(1 - 4.94 \sqrt{h} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.0021</td>
<td>0.491</td>
<td>0.0506</td>
</tr>
<tr>
<td>0.0043</td>
<td>0.282</td>
<td>0.0410</td>
</tr>
<tr>
<td>0.0064</td>
<td>0.210</td>
<td>0.0335</td>
</tr>
<tr>
<td>0.0107</td>
<td>0.109</td>
<td>0.0197</td>
</tr>
<tr>
<td>0.0150</td>
<td>0.0511</td>
<td>0.00981</td>
</tr>
<tr>
<td>0.0193</td>
<td>0.0194</td>
<td>0.00386</td>
</tr>
<tr>
<td>0.0236</td>
<td>0.00584</td>
<td>0.00118</td>
</tr>
<tr>
<td>0.0268</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>0.0293</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0327</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
A Pitot tube connected to a water-filled U-tube manometer is used to measure the total pressure within a boundary layer. Based on the data given in the table below, determine the boundary layer thickness, δ, the displacement thickness, δ*, and the momentum thickness, Θ.

From the Bernoulli equation, with

\[ \rho_{\text{air}} \ll \rho_{\text{H}_2\text{O}} \text{ it follows that} \]

\[ \rho_1 + \frac{1}{2} \rho_{\text{air}} V_1^2 = \rho_2 + \frac{1}{2} \rho_{\text{air}} V_2^2, \text{ where} \]

\[ V_1 = u, \quad V_2 = 0, \quad \rho_1 = 0, \text{ and } \rho_2 = \rho_{\text{H}_2\text{O}} h \]

Thus,

\[ u = \sqrt{\frac{2}{\rho_{\text{air}}} \frac{\delta_{\text{H}_2\text{O}} h}{N}} = \sqrt{\frac{2(9800 \text{ N/m}^2)}{1.23 \text{ kg/m}^3}} \]

or

\[ u = 126.2 \sqrt{h}, \text{ where } h \sim m, \quad u \sim \frac{m}{s} \]

For \( y > 26.8 \text{ mm we see that } h = 41.0 \text{ mm} \)

Thus,

\[ U = 126.2 \sqrt{(0.041)} = 25.55 \frac{m}{s} \]

For \( y = 23.6 \text{ mm, } u = 126.2 \sqrt{(0.0405)} \]

or

\[ \frac{u}{U} = \frac{25.40}{25.55} = 0.994 \]

Thus,

\[ \delta \approx 23.6 \text{ mm} \]

The displacement thickness, \( \delta^* \), is

\[ \delta^* = \int_0^\infty (1 - \frac{u}{U}) dy \text{ or since} \]

\[ \frac{u}{U} = \frac{126.2 \sqrt{h}}{25.55} = 4.94 \sqrt{h} \text{ this becomes} \]

\[ \delta^* = \int_0^{0.0268 \text{ m}} (1 - 4.94 \sqrt{h}) dy \]

Numerical integration of the tabulated data gives \( \delta^* = 4.18 \times 10^{-3} \text{ m} \)

(con't)
9.14 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, $\delta^*$. For air blowing past the flat plate shown in Fig. P9.14, plot the streamline A-B that passes through the edge of the boundary layer ($y = \delta_A$ at $x = 0$) at point B. That is, plot $y = y(x)$ for streamline A-B. Assume laminar boundary layer flow.

Since $Re_\delta = \frac{UL}{V} = \frac{(1.46)(4\text{ m}) \times 10^{-5}}{5 \times 10^{-5}} = 2.74 \times 10^5 < 5 \times 10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{V}{U}} \quad \text{or} \quad \delta_A = 5\left[\frac{(1.46 \times 10^{-5} \text{ m})^3 (4\text{ m})}{1 \text{ m}^2}\right]^{1/2} = 0.0382 \text{ m}$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with the plate displaced by an amount $\delta^*$. Since there is no flow through the plate or streamline A-B,

$$Q_A = Q_B, \quad \text{or} \quad U \delta_A = (\delta_B - \delta_A^*) U$$

where $\delta_A = 1.721\sqrt{\frac{V}{U}}$

or $\delta_B = 1.721\left[\frac{(1.46 \times 10^{-5} \text{ m})^3 (4\text{ m})}{1 \text{ m}^2}\right]^{1/2} = 0.01315 \text{ m}$

Thus,

$$y_A = \delta_B - \delta_A^* = 0.0382 \text{ m} - 0.01315 \text{ m} = 0.0251 \text{ m}$$

Hence, for any $x$-location

$$Q_A = Q \quad \text{or} \quad U y_A = U(y - \delta^*)$$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\frac{V}{U}}\left[\frac{(1.46 \times 10^{-5} \text{ m})^3}{1 \text{ m}^2}\right]^{1/2} = 0.0251 + 6.58 \times 10^{-3} \sqrt{\frac{V}{U}} \text{ m}$, where $x = \text{ m}$.
9.15 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.15. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, $d$, as a function of $x$ for $0 \leq x \leq 10$ ft if $U$ is to remain constant. Assume laminar flow.

For incompressible flow $Q_0 = Q(x)$ where $Q_0 = \text{flowrate into the duct}$, $A_e = UA_0 = (2 \text{ ft/s})(1 \text{ ft}^2) = 2 \frac{\text{ft}^3}{\text{s}}$

and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1 + \frac{2\delta^*}{U}$$

From Eq. (1)

$$\delta^* = 1.721 \sqrt{\frac{\nu_x}{U}} = 1.721 \left[ \frac{(1.57 \times 10^{-6} \text{ ft}^2 / \text{s}) x}{2 \frac{\text{ft}^3}{\text{s}}} \right]^{1/2} = 0.0152 \sqrt{x} \text{ ft, where} x = \text{ft}$$

Hence, from Eq. (1)

$$d = 1 + 0.0304 \sqrt{x} \text{ ft}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.
A smooth, flat plate of length \( l = 6 \text{ m} \) and width \( b = 4 \text{ m} \) is placed in water with an upstream velocity of \( U = 0.5 \text{ m/s} \). Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

\[
\delta = 5 \sqrt{\frac{\nu x}{U}} = 5 \sqrt{\frac{(1.12 \times 10^6 \text{ m}^2/\text{s}) x}{0.5 \text{ m}}} = 7.48 \times 10^{-3} \sqrt{x} \text{ m}, \text{ where } x \sim m
\]

and

\[
\tau_w = 0.332 U^{3/2} \sqrt{\frac{\mu}{x}} = 0.332 (0.5 \text{ m/s})^{3/2} \sqrt{\frac{(999 \text{ kg/m}^3)(1.12 \times 10^{-3} \text{ N s/m}^2)}{x}}
\]

\[
= \frac{0.124}{\sqrt{x}} \frac{N}{\text{m}^2}, \text{ where } x \sim m
\]

Thus, at \( x = 3 \text{ m} \)

\[
\delta = 7.48 \times 10^{-3} \sqrt{3} = 0.0130 \text{ m}
\]

\[
\tau_w = \frac{0.124}{\sqrt{3}} = 0.0716 \frac{N}{\text{m}^2}
\]

while at \( x = 6 \text{ m} \)

\[
\delta = 7.48 \times 10^{-3} \sqrt{6} = 0.0183 \text{ m}
\]

\[
\tau_w = \frac{0.124}{\sqrt{6}} = 0.0506 \frac{N}{\text{m}^2}
\]
An atmospheric boundary layer is formed when the wind blows over the earth’s surface. Typically, such velocity profiles can be written as a power law: \( u = ay^n \), where the constants \( a \) and \( n \) depend on the roughness of the terrain. As is indicated in Fig. P9.17, typical values are \( n = 0.40 \) for urban areas, \( n = 0.28 \) for woodland or suburban areas, and \( n = 0.16 \) for flat open country (Ref. 23).

(a) If the velocity is 20 ft/s at the bottom of the sail on your boat \( (y = 4 \text{ ft}) \), what is the velocity at the top of the mast \( (y = 30 \text{ ft}) \)?

(b) If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

\[
\begin{align*}
(a) \quad U &= C y^{0.16}, \quad \text{where} \quad C \quad \text{is a constant} \\
\text{Thus,} \quad \frac{U_2}{U_1} &= \left(\frac{Y_2}{Y_1}\right)^{0.16} \quad \text{or} \quad U_2 = 20 \text{ ft/s} \left(\frac{30 \text{ ft}}{4 \text{ ft}}\right)^{0.16} = 27.6 \text{ ft/s} \\
(b) \quad U &= \tilde{C} y^{0.4}, \quad \text{where} \quad \tilde{C} \quad \text{is a constant} \\
\text{Thus,} \quad \frac{U_2}{U_1} &= \left(\frac{Y_2}{Y_1}\right)^{0.40} \quad \text{or} \quad U_2 = 10 \text{ mph} \left(\frac{60 \text{ mph}}{10}\right)^{0.4} = 20.5 \text{ mph} 
\end{align*}
\]
A 30-story office building (each story is 12 ft tall) is built in a suburban industrial park. Plot the dynamic pressure $pu^{2}/2$, as a function of elevation if the wind blows at hurricane strength (75 mph) at the top of the building. Use the atmospheric boundary layer information of Problem 9.17.

From Fig. 9.17 the boundary layer velocity profile is given by $u \sim y^{0.28}$, or $u = Cy^{0.28}$, where $C$ is a constant.

Thus, $\frac{U}{U_i} = \left(\frac{y}{y_i}\right)^{0.28}$

or $u = 110 \left(\frac{y}{360}\right)^{0.28} \text{ft}$ where $y \sim \text{ft}$

Hence,

$\frac{1}{2} \rho u^2 = \frac{1}{2} \left(2.38 \times 10^{-3} \text{slug/ft}^2\right) \left[110 \left(\frac{y}{360}\right)^{0.28} \text{ft}\right]^2$

or $\frac{1}{2} \rho u^2 = 14.4 \left(\frac{y}{360}\right)^{0.56} \frac{\text{lb}}{\text{ft}^2}$, where $y \sim \text{ft}$

This is plotted in the figure below.
The typical shape of small cumulous clouds is as indicated in Fig. P9.19. Based on boundary layer ideas, explain why it is clear that the wind is blowing from right to left as indicated.

As indicated in Fig. P9.17, because of the atmospheric boundary layer the velocity of the wind generally increases with altitude. Thus, the top portions of a cloud travels faster than its base — the clouds tend to "tip" toward the direction of the wind. That is, the wind is from right to left.
9.20 Show that for any function \( f = f(\eta) \) the velocity components \( u \) and \( v \) determined by Eqs. 9.12 and 9.13 satisfy the incompressible continuity equation, Eq. 9.8.

Given \( u = U f(\eta) \), \( v = \left( \frac{\nu U}{4 \pi} \right) \left( \eta f'(\eta) - f'(\eta) \right) \)
where \( \eta = \left( \frac{U}{\nu X} \right)^{\frac{1}{2}} \) and \( \gamma = \frac{\nu}{\nu X} \)

Show that \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \) for any \( f(\eta) \).

\[ \frac{\partial u}{\partial x} = U \frac{d f'}{d \eta} = U \frac{df'}{d \eta} \left( \frac{2\eta}{\nu X} \right), \text{ where } \frac{d \eta}{d x} = -\frac{U^2 \gamma}{2 \nu X^3} x^{-\frac{2}{3}} \]

Thus,

\[ \frac{\partial u}{\partial x} = -U f'' \left[ \frac{\nu^2 \gamma}{2 \nu X^3} x^{-\frac{2}{3}} \right] = -\frac{U^2 \gamma}{2 \nu X^3} x^{-\frac{2}{3}} \frac{df''}{d \eta} \]

(1)

and

\[ \frac{\partial v}{\partial y} = \left( \frac{\nu U}{4 \pi} \right)^{\frac{1}{2}} \left[ \frac{d \eta}{d y} f' + \eta \frac{d f'}{d \eta} \right] - \frac{d \eta}{d y} f' \]

\[ = \left( \frac{\nu U}{4 \pi} \right)^{\frac{1}{2}} \left[ \eta f'' \frac{d \eta}{d y} \right] \]

\[ = \left( \frac{\nu U}{4 \pi} \right)^{\frac{1}{2}} \left[ \eta f'' \frac{d \eta}{d y} \right] , \text{ where } \frac{d \eta}{d y} = \left( \frac{\nu}{\nu X} \right)^{\frac{1}{2}} \]

Hence,

\[ \frac{\partial v}{\partial y} = \left( \frac{\nu U}{4 \pi} \right)^{\frac{1}{2}} \left( \frac{U}{\nu X} \right)^{\frac{1}{2}} \frac{df''}{d \eta} = \frac{U^2 \gamma}{2 \nu X^3} \frac{df''}{d \eta} \]

(2)

By combining Eqs. (1) and (2) we see that \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \) for any function \( f(\eta) \).
9.21 Show that by writing the velocity in terms of the similarity variable \( \eta \) and the function \( f(\eta) \), the momentum equation for boundary layer flow on a flat plate (Eq. 9.9) can be written as the ordinary differential equation given by Eq. 9.14.

The governing equations are

\[
\frac{dU}{dx} + \frac{1}{\gamma} \frac{dV}{dy} = 0 \quad \text{and} \quad \frac{dU}{dx} + \frac{1}{\gamma} \frac{dV}{dy} = \frac{\nu}{\gamma} \frac{d^2 U}{dx^2}
\]

(1)

\[
\frac{dU}{dx} + \frac{1}{\gamma} \frac{dV}{dy} = \nu \frac{d^2 U}{dx^2}
\]

(2)

Consider \( U = U f'(\eta) \) and \( V = \left( \frac{\nu U}{\nu x} \right)^{1/2} (\eta f'' - f) \) where \( f' = \frac{df}{d\eta} \) and \( \eta = \frac{U}{\nu x} \).

Thus, \( \frac{d\eta}{\gamma} = -\frac{1}{2} \frac{\nu}{\nu x} \gamma x^{-2} \) and \( \frac{d\eta}{\gamma} = \frac{\nu}{\gamma x^2} \).

(3)

so that

\[
\frac{d^2 U}{dx^2} = \frac{d}{dx} \left( U f' \right) = U \frac{d^2 f'}{d\eta^2} = U \frac{d^2 f'}{d\eta^2} \frac{d\eta}{\gamma} = -\frac{1}{2} \frac{\nu}{\nu x} \eta f''
\]

(4)

\[
\frac{dV}{dy} = \left( \frac{\nu U}{\nu x} \right)^{1/2} \left( \eta f'' - f \right) = \left( \frac{\nu U}{\nu x} \right)^{1/2} \left( \eta f'' + f' - f \right) \frac{d\eta}{\gamma} = \left( \frac{\nu U}{\nu x} \right)^{1/2} \eta f'' + \frac{\nu U}{\nu x} f''
\]

(5)

Thus, by using Eqs. (4) and (5) we see that Eq. (1) is satisfied for any function \( f(\eta) \).

Also, \( \frac{d^2 f}{d\eta^2} = \frac{d}{d\eta} \left( U f' \right) \left[ \frac{\nu U}{\nu x} x^{1/2} \right] = \left( \frac{\nu U}{\nu x} \right)^{1/2} f''
\)

(6)

and

\[
\frac{d^2 f}{d\eta^2} = \left( \frac{\nu U}{\nu x} \right)^{1/2} f'' = \left( \frac{\nu U}{\nu x} \right)^{1/2} f'' = \frac{\nu U}{\nu x} f''
\]

(7)

Thus, by using Eqs. (2.1), (6), and (7) with Eq. (2.2) we obtain

\[
(U f')(\frac{\nu U}{\nu x} \eta f'') + \left( \frac{\nu U}{\nu x} \right)^{1/2} (\eta f' - f) \left( \frac{\nu U}{\nu x} \right)^{1/2} f'' = \nu \frac{\nu U}{\nu x} f''
\]

which simplifies to

\[
2 f'' - f f'' = 0
\]

From Eq. (2.1) the boundary conditions at \( y = 0 \) (i.e., \( \eta = 0 \)) become

\[
u = 0 = U f(0) \quad \text{and} \quad \nu = 0 = \left( \frac{\nu U}{\nu x} \right)^{1/2} (0 f(0) - f(0))
\]

That is, \( f(0) = 0 \) and \( f'(0) = 0 \).

Similarly, as \( y \to \infty \) (i.e., \( \eta \to \infty \)) we require \( U \to U \). Thus, from

Eq. (2.1) \( f' \to 1 \) as \( \eta \to \infty \).
An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of \( \text{Re}_{cr} = 5 \times 10^5 \). If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

**At 10,000 ft:**

\[
\text{Re}_{cr} = \frac{U x_{cr}}{V}, \quad \text{where} \quad U = 400 \text{ mph} \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) = 587 \frac{\text{ft}}{s}
\]

and from Table C.1, \( V = \frac{U}{C} = \frac{3.534 \times 10^{-7} \text{ lb} \cdot s}{1.756 \times 10^{-3} \text{ s} \cdot \text{ lb} \cdot \text{ ft}^2} = 2.01 \times 10^{-4} \frac{\text{ft}^2}{s} \)

\[
\text{Hence, with } \text{Re}_{cr} = 5 \times 10^5,
\]

\[
x_{cr} = \frac{V \text{Re}_{cr}}{U} = \left( \frac{2.01 \times 10^{-4} \frac{\text{ft}^2}{s}}{587 \frac{\text{ft}}{s}} \right) (5 \times 10^5) = 0.171 \text{ ft}
\]

**At sea-level:**

\[
\text{Re}_{cr} = \frac{U x_{cr}}{V}, \quad \text{where} \quad U = 400 \text{ mph} \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) \left( \frac{5280 \text{ ft}}{\text{mi}} \right) = 587 \frac{\text{ft}}{s}
\]

\[
\text{and} \quad V = 1.57 \times 10^{-4} \frac{\text{ft}^2}{s}
\]

\[
\text{Hence},
\]

\[
x_{cr} = \frac{V \text{Re}_{cr}}{U} = \left( \frac{1.57 \times 10^{-4} \frac{\text{ft}^2}{s}}{587 \frac{\text{ft}}{s}} \right) (5 \times 10^5) = 0.134 \text{ ft}
\]

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.
A laminar boundary layer velocity profile is approximated by \( u/U = [2 - (y/\delta)][y/\delta] \) for \( y < \delta \), and \( u = U \) for \( y > \delta \). (a) Show that this profile satisfies the appropriate boundary conditions. (b) Use the momentum integral equation to determine the boundary layer thickness, \( \delta - \delta(y) \).

\[
\frac{u}{U} = g(Y) = 2Y - Y^2 \text{ where } Y = y/\delta
\]

Thus, \( \left. \frac{u}{U} \right|_{y=0} = 0 \) as it must, \( \left. \frac{u}{U} \right|_{y=\delta} = 2 - 1 = 1 \) or \( u=U \) at \( y=\delta \) as it must.

Also, \( \frac{du}{dy} = U\left[ \frac{2}{\delta} - \frac{2Y}{\delta^2} \right] \) so that \( \left. \frac{du}{dy} \right|_{y=\delta} = U\left[ \frac{2}{\delta} - \frac{2}{\delta} \right] = 0 \)

(b) From the momentum integral equation,

\[
\delta = \sqrt{\frac{2C_2 \nu x}{\nu C_1}}, \text{ where } C_1 = \int_0^1 g(1-y) \, dy \text{ and } C_2 = \frac{\partial \rho}{\partial Y} \bigg|_{Y=0}
\]

Thus,

\[
C_1 = \int_0^1 (2Y - Y^2)(1-2Y + Y^2) \, dY = \int_0^1 (2Y - 5Y^2 + 4Y^3 - Y^4) \, dY
\]

\[
= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{5}
\]

and

\[
C_2 = \left. (2 - 2Y) \right|_{Y=0} = 2
\]

so that

\[
\delta = \sqrt{\frac{2(2) \nu x}{\nu C_1}} = \sqrt{\frac{30 \nu x}{\nu}}
\]

Hence, with \( Re_x = \frac{UX}{\nu} \),

\[
\frac{\delta}{x} = \sqrt{\frac{30}{U \nu Re_x}} = \frac{5.48}{U \sqrt{Re_x}}
\]
A laminar boundary layer velocity profile is approximated by the two straight-line segments indicated in Fig. P9.25. Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$, and wall shear stress, $\tau_w = \tau_w(x)$. Compare these results with those in Table 9.2.

\[ \delta = \sqrt{\frac{\frac{2}{3} \nu x}{U \gamma_1}}, \quad \text{where} \quad \gamma_1 = \int_0^1 g(1-g)\,dY \quad \text{and} \quad \gamma_2 = \frac{dg}{dy} \bigg|_{Y=0} \]  
\[ \text{and} \quad \frac{u}{U} = g(Y) \quad \text{with} \quad Y = \frac{y}{\delta}, \]

For $0 \leq Y < \frac{1}{2}$, $g = a_1 + b_1 Y$ with the constants $a_1$ and $b_1$ obtained from $g = \frac{3}{5}$ at $Y = \frac{1}{2}$ and $g = 0$ at $Y = 0$. Thus, $a_1 = 0$, $b_1 = \frac{3}{5}$.

\[ g = \frac{3}{5} Y \quad \text{for} \quad 0 \leq Y < \frac{1}{2} \]

Hence, $\gamma_2 = \frac{3}{5}$

Similarly, for $\frac{1}{2} \leq Y \leq 1$, $g = a_2 + b_2 Y$ with $g = \frac{3}{5}$ at $Y = \frac{1}{2}$ and $g = 1$ at $Y = 1$.

Thus, $\frac{2}{5} = a_2 + \frac{1}{2} b_2$ and $1 = a_2 + b_2$ which give $a_2 = \frac{1}{5}$, $b_2 = \frac{2}{5}$.

\[ g = \frac{1}{5} + \frac{2}{5} Y \quad \text{for} \quad \frac{1}{2} \leq Y < \frac{3}{5}, \]

Hence, $\gamma_1 = \int_0^1 g(1-g)\,dY = \int_0^{\frac{5}{24}} Y(1-\frac{5}{2} Y)\,dY + \int_{\frac{5}{24}}^1 (\frac{5}{24} + \frac{3}{5} Y)(1-\frac{5}{2} Y)\,dY$

\[ = \frac{5}{4} \int_0^{\frac{5}{24}} (3Y-4Y^2)\,dY + \frac{5}{4} \int_{\frac{5}{24}}^1 (1+2Y)(1-Y)\,dY \quad \text{which upon integration gives} \]

\[ \gamma_1 = 0.1574 \]  

By combining Eqs. (1), (2) and (3) we obtain
\[ \delta = \left[ \frac{2 (\frac{3}{5}) \nu x}{0.1574 U} \right] = 4.12 \sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta \sqrt{\frac{U}{\nu}} = \frac{4.12}{\sqrt{\frac{U}{\nu}}} \]

Also, $\frac{\nu U}{\delta} C_2 = \frac{4 \mu U}{3 \delta}$ or $C_f = \frac{\sqrt{2 \gamma_1 C_2}}{\sqrt{\frac{\nu}{\nu x}}} = \frac{\sqrt{2 (0.1574) (\frac{3}{5})}}{\sqrt{\frac{\nu}{\nu x}}} = 0.648 \sqrt{\frac{\nu}{\nu x}}$

Compare these results to those in Table 9.2.
9.26* An assumed dimensionless laminar boundary layer profile for flow past a flat plate is given in the table below. Use the momentum integral equation to determine \( \delta = \delta(x) \). Compare your result with the exact Blasius solution result (see Table 9.2).

From the momentum integral equation

\[
\delta = \sqrt{\frac{2C_2 \nu \chi}{U C_1}}, \quad \text{where} \quad C_2 = \left. \frac{\partial q}{\partial Y} \right|_{Y=0}
\]

and

\[
C_1 = \int_0^1 g(1-g) \, dY \quad \text{with} \quad \frac{d}{dY} = g(Y) \quad \text{and} \quad Y = \frac{Y}{\delta}
\]

The value of \( C_2 \) can be approximated as

\[
C_2 \approx \frac{\Delta \left( \frac{Y}{\delta} \right)}{\Delta \left( \frac{Y}{\delta} \right)} \bigg|_{Y=0} = \frac{0.133}{0.080} = 1.66
\]

and the value of \( C_1 \) can be obtained from numerical integration.

<table>
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<th>( Y )</th>
<th>( g(1-g) )</th>
</tr>
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</tr>
</tbody>
</table>

The result of the integration is

\[
C_1 = \int_0^1 g(1-g) \, dY \approx 0.131 \quad \text{so that}
\]

\[
\delta = \left[ \frac{2V \chi (1.66)}{U (0.131)} \right]^\frac{1}{2} = 5.03 \left( \frac{V \chi}{U} \right)^\frac{1}{2}
\]

or

\[
\frac{\delta}{\chi} = \frac{5.03}{\sqrt{Re_x}}, \quad \text{where} \quad Re_x = \frac{pV \chi}{\mu}
\]

Note: The Blasius solution has \( \delta \), not \( 5.03 \).
For a fluid of specific gravity $SG = 0.86$ flowing past a flat plate with an upstream velocity of $U = 5$ m/s, the wall shear stress on a flat plate was determined to be as indicated in the table below. Use the momentum integral equation to determine the boundary layer momentum thickness, $\Theta = \Theta(x)$ Assume $\Theta = 0$ at the leading edge, $x = 0$.

$$\tau_w = \rho U^2 \frac{d\Theta}{dx}$$

it follows that

$$d\Theta = \frac{\tau_w}{\rho U^2} \, dx$$

which can be integrated to give (using $\Theta = 0$ at $x = 0$)

$$\Theta = \frac{1}{\rho U^2} \int_0^x \tau_w \, dx = \frac{1}{0.86 \times (1000 \text{ kg/m}^3) (5 \text{ m/s})^2} \int_0^x \tau_w \, dx$$

or

$$\Theta = 4.65 \times 10^{-5} \int_0^x \tau_w \, dx , \text{ where } \Theta \sim m , \ x \sim m , \text{ and } \tau_w \sim \frac{N}{m^2}$$

For $0 \leq x \leq 2.0$ m, integrate Eq. (1) to determine $\Theta$ as a function of $x$. To do so, we need the value of $\tau_w$ at $x = 0$, which is not given in the table. Theoretically, $\tau_w = \infty$ at the leading. For our purposes, based on the extrapolated curve below, assume $\tau_w = 22 \frac{N}{m^2}$ at $x = 0$.
9.28 The square flat plate shown in Fig. P9.28a is cut into four equal-sized pieces and arranged as shown in Fig. P9.28b. Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow. Explain your answer physically.

For case (a):
\[ \Delta F_a = \frac{1}{2} \rho U^2 C_{D_f} A \] where \( C_{D_f} = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{U_0}} \text{ and } A = l^2 \)
Thus,
\[ \Delta F_a = \frac{1}{2} \rho U^2 \frac{1.328\sqrt{U}}{U_0} l^2 = 0.664 \rho U^{0.5} \sqrt{U} l^{3.5} \] (1)

For case (b):
\[ \Delta F_b = \frac{1}{2} \rho U^2 C_{D_f} A \] where \( C_{D_f} = \frac{1.328}{U(4L)} \text{ and } A = (4L)(\frac{l}{4}) = l^2 \)
Thus,
\[ \Delta F_b = \frac{1}{2} \rho U^2 \frac{1.328\sqrt{U}}{4U_0} l^2 = \frac{1}{2} \big( 0.664 \rho U^{0.5} \sqrt{U} l^{3.5} \big) \] (2)

By comparing Eqs. (1) and (2) we see that
\[ \frac{\Delta F_a}{\Delta F_b} = \frac{2.0}{\Delta F_b} \]

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.
9.29 The two flat plates shown in Fig. P9.29 are to have the same drag. Determine the upstream velocity \( U_a \) in terms of \( U_b \) and \( n \). Assume laminar flow. Explain your answer physically.

\[ \theta_a = \theta_b \]

or

\[ \frac{1}{2} \rho U_a^2 A_a C_{D_a} = \frac{1}{2} \rho U_b^2 A_b C_{D_b} \]

where \( A_a = A_b = (n\ell)l \) and

\[ C_D = \frac{1.328}{\sqrt{Re}} \quad \text{or} \quad C_{D_a} = \frac{1.328}{\sqrt{Re_a}} \quad C_{D_b} = \frac{1.328}{\sqrt{Re_b}} \]

Thus, Eq. (1) gives

\[ U_a^2 C_{D_a} = U_b^2 C_{D_b} \]

or

\[ \frac{U_a^2}{\sqrt{Re_a}} = \frac{U_b^2}{\sqrt{Re_b}} \quad \text{where} \quad Re_a = \frac{U_a l}{v} \quad \text{and} \quad Re_b = \frac{U_b (n\ell)}{v} \]

Hence,

\[ \frac{U_a^2}{\sqrt{U_a l/v}} = \frac{U_b^2}{\sqrt{U_b n\ell/v}} \]

or

\[ U_a^{3/2} = U_b^{3/2} / n^{1/2} \]

Thus,

\[ U_b = n^{1/2} U_a \]
If the drag on one side of a flat plate parallel to the upstream flow is \( D \) when the upstream velocity is \( U \), what will the drag be when the upstream velocity is \( 2U \); or \( U/2 \)? Assume laminar flow.

For laminar flow, \( D = \frac{1}{2} \rho U^2 C_{Df} A \), where \( C_{Df} = \frac{1.328}{\frac{U}{V}} \).

Thus, \( D = \frac{1}{2} \rho U^2 \frac{1.328 V}{4UV} A = 0.664 \rho A \frac{V}{U} U^{3/2} \sim U^{3/2} \).

Hence, \( \frac{D}{D_{U}} = \frac{U^{3/2}}{(2U)^{3/2}} \) or \( \frac{D_{U}}{D_{U}^{2}} = 2.83 \frac{D}{D_{U}} \).

And \( \frac{D_{U}}{D_{U/2}} = \frac{U^{3/2}}{(U/2)^{3/2}} \) or \( \frac{D_{U/2}}{D_{U/2}} = 0.354 \frac{D_{U}}{D_{U/2}} \).
Air flows past a parabolic-shaped flat plate oriented parallel to the free stream shown in Fig. P9.31. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar flow.

Treat each strip of thickness \( dy \) and length \( l = \xi(y) \) as a small flat plate with drag \( d\Theta \) where for laminar flow

\[
d\Theta = C_{Df} \frac{1}{2} \rho U^2 dA \quad \text{with} \quad dA = l \, dy \quad \text{and} \quad C_{Df} = \frac{1.328}{\sqrt{\text{Re}_h}} = \frac{1.328}{\sqrt{\frac{UL}{\mu}}}
\]

Thus,

\[
d\Theta = \frac{1.328}{\sqrt{\frac{UL}{\mu^2}}} \frac{1}{2} \rho U^2 l \, dy = 0.664 \sqrt{\frac{\mu}{\rho}} U^{3/2} \sqrt{\frac{l}{\mu}} \, dy
\]

But \( l = 4 - y^2 \) so that

\[
\Theta = \int d\Theta = \int_{-2}^{2} 0.664 \sqrt{\frac{\mu}{\rho}} U^{3/2} \sqrt{\frac{4 - y^2}{\mu}} \, dy
\]

\[
= 2(0.664) \sqrt{\frac{\mu}{\rho}} U^{3/2} \int_{0}^{2} \sqrt{\frac{4 - y^2}{\mu}} \, dy
\]

\[
= 1.328 \sqrt{\frac{\mu}{\rho}} U^{3/2} \left[ \frac{y\sqrt{4-y^2}}{2} + 4 \sin^{-1}\left(\frac{y}{2}\right) \right]_0^2
\]

\[
= 1.328 \sqrt{\frac{\mu}{\rho}} U^{3/2} \left[ \frac{4 \sin^{-1}(1)}{\frac{1}{2}} \right] \left( \frac{1}{2} \right)
\]

\[
= 1.328 \sqrt{\frac{\mu}{\rho}} U^{3/2} (2\pi)(\frac{1}{2}) \quad \text{Note: The units on the integral are ft}^{3/2} \quad \text{(i.e. 2\pi ft}^{3/2})
\]

Thus,

\[
\Theta = 1.328 \left[ 3.74 \times 10^{-7} \frac{lb \cdot s}{\text{ft}^2} \left( 2.38 \times 10^{3} \text{slug} \right) \right] \left( \frac{2}{\pi} \right)^{3/2} \left( \frac{2\pi ft^{3/2}}{ft^{3/2}} \right)
\]

\[
= 6.47 \times 10^{-4} \text{lb}
\]
9.32 It is relatively easy to design an efficient nozzle to accelerate a fluid. Conversely, it is very difficult to build an efficient diffuser to decelerate a fluid without boundary layer separation and its subsequent inefficient flow behavior. Use the ideas of favorable and adverse pressure gradients to explain these facts.

For a nozzle as shown, the pressure decreases in the direction of flow and $\frac{dV}{dx} > 0$. Thus the boundary layer fluid flows in a region of favorable pressure gradient, $\frac{dP}{dx} < 0$. This reduces the chance of boundary layer separation. The fluid follows the nozzle contour with little difficulty, regardless of its shape.

For a diffuser as shown, the pressure increases in the direction of flow and $\frac{dV}{dx} < 0$. Thus the boundary layer fluid flows in a region of adverse pressure gradient, $\frac{dP}{dx} > 0$. If the pressure gradient is too large (i.e., the diffuser angle too large), the boundary layer will separate and the diffuser will not function properly.
9.33 Two small holes are drilled opposite each other in a circular cylinder as shown in Fig. 9.33. Thus, when air flows past the cylinder, air will circulate through the interior of the cylinder at a rate of \( Q = K(p_1 - p_2) \), where the constant \( K \) depends on the geometry of the passage connecting the two holes. It is assumed that the flow around the cylinder is not affected by either the presence of the two holes or the small flow rate through the passage. Let \( Q_0 \) denote the flow rate when \( \theta = 0 \). Plot a graph of \( Q/Q_0 \) as a function of \( \theta \) for \( 0 \leq \theta \leq \pi/2 \) if (a) the flow is inviscid, and (b) if the boundary layer on the cylinder is turbulent (see Fig. 9.17c for pressure data).

\[ C_p = \frac{p - p_0}{\frac{1}{2} \rho U^2} = 1 - 4\sin^2 \theta \]

Thus, \( Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)] = K\left[\frac{1}{2} \rho U^2 (1 - 4\sin^2 \theta) - \frac{1}{2} \rho U^2 (1 - 4\sin^2 (\theta + \pi))\right] \)

but \( \sin^2 \theta = \sin^2 (\theta + \pi) \)

Hence, \( Q = 0 \) for inviscid flow. Note: This is to be expected because of the symmetrical pressure distribution.

(b) For a turbulent boundary layer:

\[ Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)] = \frac{1}{2} \rho U^2 K[C_{p1} - C_{p2}] \]

where \( C_{p1} \) is for \( \theta \) and \( C_{p2} \) is for \( 180 - \theta \) deg.

Obtain \( C_p \) data from Fig. 9.17.

Note: \( C_p = 1 \) for \( \theta = 0 \) and \( C_p \approx 0.4 \) for \( \theta = 180 \) deg.

Thus, \( Q_0 = Q \bigg|_{\theta=0} = \frac{1}{2} \rho U^2 K[1 - (-0.4)] = 1.4\left(\frac{1}{2} \rho U^2 K\right) \)

so that

\[ \frac{Q}{Q_0} = \frac{1}{1.4}\left(\frac{1}{2} \rho U^2 K\right) \]

The results are tabulated and plotted below.

\[(\text{cont'})\]
<table>
<thead>
<tr>
<th>$\theta$, deg</th>
<th>180 - $\theta$, deg</th>
<th>$C_{p1}$</th>
<th>$C_{p2}$</th>
<th>$Q/Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>180</td>
<td>1.00</td>
<td>-0.40</td>
<td>1.00</td>
</tr>
<tr>
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<td>165</td>
<td>0.70</td>
<td>-0.40</td>
<td>0.79</td>
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<td>75</td>
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<td>-0.57</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>-1.90</td>
<td>-1.90</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Diagram:**

$Q/Q_0$ vs $\theta$

- $Q/Q_0$ on the y-axis
- $\theta$, deg on the x-axis
Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.34. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.

\[ \tau_w \, dA \]

where \( \tau_w = 0.332 \, U^{3/2} \sqrt{\frac{\rho \mu}{x}} \)

Thus,
\[ \delta = \int \tau_w \, dA \]

\[ \delta = 0.332 \, U^{3/2} \sqrt{\rho \mu} \int_{x=0}^{0.5} \int_{y=0}^{0.5-x} dy \, dx \]

\[ = 0.332 \, U^{3/2} \sqrt{\rho \mu} \int_{x=0}^{0.5} \frac{0.5-x}{x} \, dx \]

\[ = 0.332 \, U^{3/2} \sqrt{\rho \mu} \left[ \frac{0.5}{2} \frac{x^2}{x} \bigg|_{x=0}^{0.5} - \frac{2}{3} \frac{x^{3/2}}{x} \right]_{0}^{0.5} \]

\[ = 0.664 \left( 0.2 \, \frac{m}{s} \right)^{3/2} \sqrt{999 \, \frac{kg}{m^3} \left( 1.12 \times 10^{-6} \frac{N \cdot s}{m^2} \right)} \left[ \sqrt{0.5} - \frac{2}{3} (0.5)^{3/2} \right] \]

or

\[ \delta = 0.0296 \, N \]
A ceiling fan consists of five blades of 0.80-m length and 0.10-m width which rotate at 100 rpm. Estimate the torque needed to overcome the friction on the blades if they act as flat plates.

Let \( dM \) = torque from the drag on element

or \( dA \) of the blade

\[
dM = (\alpha_{\text{top}} + \alpha_{\text{bottom}}) y = 2(\frac{1}{2} \rho U^2 C_D \, dA) y
\]

where \( U = \omega y \) and \( \omega = 100 \, \text{rev} / \text{min} \times \frac{1 \text{rev}}{60 \text{s}} \times \frac{2\pi \text{rad}}{1 \text{rev}} \)

or \( \omega = 10.47 \, \text{rad/s} \)

The maximum \( \text{Re}_x \) will occur at point (1) where \( y = L \) or

\[
\text{Re}_x = \frac{U \, L}{v} = \frac{U \ell}{v} = \frac{(10.47 \, \text{rad/s})(0.8 \, \text{m})(0.1 \, \text{m})}{1.46 \times 10^{-5} \, \text{m}^3 / \text{s}} = 5.7 \times 10^5
\]

Thus, at all points on the blade \( \text{Re}_x < \text{Re}_{x,cr} = 5 \times 10^5 \) and the flow is laminar.

\[
C_D = \frac{1.328}{\sqrt{\text{Re}_x}} = \frac{1.328 V}{\sqrt{U \ell}}
\]

so that from Eq. (1)

\[
dM = \rho U^2 \frac{1.328 V}{\sqrt{U \ell}} (2 \, dy) y = 1.328 \rho U^2 \frac{3}{2} YY^2 \, dy 
\]

or

\[
dM = 0.0669 \, y^{3/2} \, dy \quad N \cdot m 
\]

Thus, the net torque on the four blades is

\[
M = 5 \int_{0.8 \, \text{m}}^{0.8 \, \text{m}} 0.0669 \, y^{3/2} \, dy = 5(0.0669) \left( \frac{2}{3} \right) y^{3/2} \bigg|_{0}^{0.8} 
\]

or

\[
M = 0.0438 \, N \cdot m
\]
As shown in Video V9.2 and Fig. P9.37a, a kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in Fig. P9.37b. Comment on reasons why the two sets of values may differ.

For a flat plate \( \mathcal{D} = \frac{1}{2} \rho U^2 C_{Df} A \) where \( A = 17 \text{ft}(2 \text{ft}) = 34 \text{ ft}^2 \) and \( C_{Df} \) is a function of \( Re_f = \frac{UL}{\nu} \)

Thus,

\[
Re_f = \frac{17 U}{\frac{1.2 \times 10^{-4}}{1.40 \times 10^6} U} = 1.40 \times 10^6 U
\]

Consider \( 1 \leq U \leq \frac{8}{3} \) ft/s, or \( 1.40 \times 10^6 \leq Re_f \leq 1.12 \times 10^7 \)

From Fig. 9.15 we see that in this \( Re_f \) range the boundary layer flow is in the transitional range. Thus, from Table 9.3

\[
C_{Df} = 0.455 / ((\log Re_f)^{2.58} - 1700 / Re_f)
\]

By combining Eqs. (1), (2), and (3):

\[
\mathcal{D} = \frac{1}{2} \left( \frac{1.94 \text{slug}}{\text{ft}^3} \right) U^2 C_{Df} (34 \text{ft}^2)
\]

or

\[
\mathcal{D} = 33.0 U^2 \left[ \frac{0.455 / ((\log (1.40 \times 10^6 U)^{2.58} - 1700 / (1.40 \times 10^6 U)]}{1}
\]

The results from this equation are plotted below.

\[
\begin{array}{c|c}
\mathcal{U}, \text{ ft/s} & \mathcal{D}, \text{ lb} \\
1 & 0.0986 \\
2 & 0.410 \\
3 & 0.909 \\
4 & 1.58 \\
5 & 2.42 \\
6 & 3.43 \\
7 & 4.59 \\
8 & 5.90 \\
\end{array}
\]

\[ \text{FIGURE P9.37} \]
A sphere of diameter $D$ and density $\rho_s$ falls at a steady rate through a liquid of density $\rho$ and viscosity $\mu$. If the Reynolds number, $Re = \rho DU/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18U$.

For steady flow $\sum F_z = 0$

or $D + F_B = W$, where $F_B = \text{buoyant force} = \rho_s g V = \rho_s g \left(\frac{4}{3}\right) \pi \left(\frac{D}{2}\right)^3$

$W = \text{weight} = \rho g V = \rho g \left(\frac{4}{3}\right) \pi \left(\frac{D}{2}\right)^3$

and $D = \text{drag} = C_D \frac{1}{2} \rho DU^2$, or since $Re < 1$

$D = 3\pi DU \mu$

Thus,

$3\pi DU \mu + \rho g \left(\frac{4}{3}\right) \pi \left(\frac{D}{2}\right)^3 = \rho_s g \left(\frac{4}{3}\right) \pi \left(\frac{D}{2}\right)^3$

which can be rearranged to give

$\mu = \frac{g D^2 (\rho_s - \rho)}{18 U}$
9.39 Determine the drag on a small circular disk of 0.01 ft diameter moving 0.01 ft/s through oil with a specific gravity of 0.87 and a viscosity 10,000 times that of water. The disk is oriented normal to the upstream velocity. By what percent is the drag reduced if the disk is oriented parallel to the flow?

\[ D = C_D \frac{1}{2} \rho U^2 A \], where \( \rho = (0.87)\left(1.94 \frac{slugs}{ft^3}\right) = 1.688 \frac{slugs}{ft^3} \)

and

\[ \mu = 10^4 \mu_{H_2O} = 10^4 \left(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right) = 0.234 \frac{lb \cdot s}{ft^2} \]

Thus, \( Re = \frac{UD}{\nu} = \frac{\rho UD}{\mu} = \frac{(1.688 \frac{slugs}{ft^3})(0.01 ft^2)(0.01 ft)}{0.234 \frac{lb \cdot s}{ft^2}} = 7.21 \times 10^{-4} \ll 1 \)

so that the low Re data of Table 9.4 is valid.

For the disk normal to the flow, \( C_D = \frac{20.4}{Re} = \frac{20.4}{7.21 \times 10^{-4}} = 28,300 \)

so that from Eq. (1)

\[ D = 28,300 \left(\frac{1}{2}\right)(1.688 \frac{slugs}{ft^3})(0.01 ft^2)^2 = 1.88 \times 10^{-4} lb \]

If the disk is parallel to the flow, \( C_D = \frac{13.6}{Re} \) so that

\[ \frac{D_{parallel}}{D_{normal}} = C_{D_{parallel}} \left(\frac{13.6}{20.4}\right) = 0.667 \text{, a 33.3% reduction} \]

9.40 For small Reynolds number flows the drag coefficient of an object is given by a constant divided by the Reynolds number (see Table 9.4). Thus, as the Reynolds number tends to zero, the drag coefficient becomes infinitely large. Does this mean that for small velocities (hence, small Reynolds numbers) the drag is very large? Explain.

For a given object \( C_D = \frac{C}{Re} \) (where the value of \( C \) depends on the shape of the object), provided \( Re \ll 1 \). Thus, as \( Re \to 0 \), \( C_D \to \infty \).

However,

\[ D = C_D \frac{1}{2} \rho U^2 A = \frac{C}{(\mu U)} \frac{1}{2} \rho U^2 A \sim U \]

That is, as \( U \to 0 \) (i.e., \( Re \to 0 \)), then \( D \sim U \)

Thus, does \( C_D \to \infty \) mean that \( D \to \infty \)? No.
9.41 A small spherical water drop of diameter 0.002 in. exists in the atmosphere at 5000 ft altitude. Will the drop rise or fall if it is in a thermal (an upward flowing column of air) having a speed of 4 ft/s? Repeat for speeds of 1 ft/s and 0.1 ft/s.

In stationary air the particle falls with speed \( U \) such that \( D + F_B = W \), where

if \( Re = \frac{UD}{\nu} < 1 \) then

\[ D = \text{drag} = 3\pi DU\mu \]

Also, \( W = \gamma_h \frac{4\pi}{3} \left( \frac{D}{2} \right)^3 = \text{weight} \)

and \( F_B = \gamma_h \frac{4\pi}{3} \left( \frac{D}{2} \right)^3 = \text{buoyant force} \)

Since \( \nu_{\text{air}} \ll \nu_h \) we can neglect the buoyant force.

That is, \( 3\pi DU\mu = \gamma_h \frac{4\pi}{3} \left( \frac{D}{2} \right)^3 \) or \( U = \frac{\gamma_h D^2}{18\mu} \)

At an altitude of 5000 ft, \( \mu = 3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2} \), so that

\[ U = \left( \frac{62.4}{18} \right) \left( \frac{0.002 ft}{12 ft} \right)^2 = 0.265 \frac{ft}{s} \]

Thus, the drop will rise if the upward velocity is \( 4 \frac{ft}{s} \) or \( 1 \frac{ft}{s} \), but it will fall if it is \( 0.1 \frac{ft}{s} \).

**Note:** The above is correct if \( Re < 1 \). Since \( Re = \frac{UD}{\nu} \) or \( Re = \left( \frac{2.048 \times 10^{-3} \frac{lb}{ft^2}}{3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}} \right) \left( 0.265 \frac{ft}{s} \right) \left( \frac{0.002 ft}{12 ft} \right) = 0.249 \) the low \( Re \) drag equation, Eq.(1), is valid.
A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.

For steady rise \( \Sigma F_B = 0 \)

or

\[
F_B = W + \mathcal{D}, \text{ where } \mathcal{D} = \text{drag} = C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2
\]

\[W = \text{weight} = 0.0245 N\]

\[F_B = \text{buoyant force} = g V = g \frac{4}{3} \pi \left( \frac{D}{2} \right)^3\]

Thus,

\[
g \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 = W + C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2
\]

or

\[
(9.80 \times 10^3 N/m^2) \frac{4}{3} \pi \left( \frac{0.0381}{2} \right)^3 = 0.0245 + \frac{1}{2} C_D (999 \text{ kg/m}^3) U^2 \frac{\pi}{4} (0.0381 m)^3
\]

or

\[C_D U^2 = 0.455, \text{ where } U \sim \frac{m}{s}\]  \hspace{1cm} (1)

Also, \(Re = \frac{U D}{\nu}\)

or

\[Re = \frac{U (0.0381 m)}{1.12 \times 10^{-6} \text{ m}^2/\text{s}} = 3.40 \times 10^4 U, \text{ where } U \sim \frac{m}{s}\]  \hspace{1cm} (2)

Finally, from Fig. 9.21: \(C_D\)

\[
\text{Re} = \begin{cases} \text{ } & \text{ } \\ \text{ } & \text{ } \end{cases}
\]

Trial and error solution: Assume \(C_D\); obtain \(U\) from Eq. (1), \(Re\) from Eq. (2); check \(C_D\) from Eq. (3), the graph.

Assume \(C_D = 0.5\) \(\rightarrow U = 0.954 \frac{m}{s} \rightarrow Re = 3.24 \times 10^4 \rightarrow C_D = 0.4 \neq 0.5\)

Assume \(C_D = 0.4\) \(\rightarrow U = 1.06 \frac{m}{s} \rightarrow Re = 3.62 \times 10^4 \rightarrow C_D = 0.4 \text{ (checks)}\)

Thus, \(U = 1.06 \frac{m}{s}\)

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.
A 3-m-diameter meteorological balloon that weighs 6 N when deflated is filled with helium having \( \gamma = 1.63 \) N/m\(^3\). Plot a graph of the rate at which it rises as a function of altitude for altitudes from sea level to its maximum altitude. Assume standard atmospheric conditions (Table C.2) and that the diameter remains constant.*

For steady rise \( \sum F_x = 0 \) or \( F_B = W + \rho \beta + W_{He} \)

where \( \Delta \gamma = C_0 \frac{1}{2} \rho U^2 \frac{4}{3} \pi D^3 \) and \( F_B = \gamma \Delta \gamma = \rho g \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 \)

Thus,

\[
\rho g \frac{4}{3} \pi \left( \frac{3}{2} \right) = 6 N + C_0 \frac{1}{2} \rho U^2 \frac{4}{3} \pi (3 m)^2 + (1.63 \frac{m}{m^3}) \frac{4}{3} \pi \left( \frac{3}{2} m \right)^3
\]

or

\[
C_0 U^2 = 400g - \frac{8.22}{g}, \text{ where } U \approx \frac{m}{s}, \rho \approx \frac{kg}{m^3}
\]

Also, \( Re = \frac{\rho U D}{\mu} \)

or \( Re = \frac{3 \rho U}{\mu}, \text{ where } \mu \approx \frac{N s}{m^2} \)

Finally, from Fig. 9.23 \( C_0 \)

\[
Re
\]

For a given altitude obtain \( \rho, \mu \) from Table C.2. Then a trial and error solution for \( U \): Assume \( C_0 \); obtain \( U \) from Eq. (1), \( Re \) from Eq. (2); check \( C_0 \) from Eq. (3), the graph.

Note: The maximum altitude will occur when \( U = 0 \) (i.e., \( \beta = 0 \) so that \( F_B = W + W_{He} \)). From Eq. (3) this gives \( \rho = \frac{2.06}{g} = 0.21 \frac{kg}{m^3} \)

This occurs at an altitude of approximately \( z = 15,000 m \).

Since Fig. 9.23 is not in equation form, combine a computer/graph solution: a) select an altitude \( z < 15,000 m \); b) lookup \( \rho, \mu \) (see Table C.2); c) assume a value of \( C_0 \) and calculate \( U \) from Eq. (1); d) calculate \( Re \) from Eq. (2) and lookup \( C_0 \) in Fig. 9.23; e) compare new \( C_0 \) value with the assumed one — iterate until they agree; f) back to step a. The results are shown below.

* In an actual weather balloon the diameter increases considerably as it rises (i.e., as the surrounding pressure decreases.)

(Con't)

9-41
z, m  | U, m/s
---|---
0   | 11.41
1000 | 11.76
2000 | 11.62
3000 | 11.71
4000 | 11.78
5000 | 11.55
6000 | 11.56
7000 | 11.52
8000 | 11.42
9000 | 11.25
10000 | 10.97
11000 | 10.83
12000 | 10.47
13000 | 9.59
14000 | 6.72
15000 | 0
A 500-N cube of specific gravity $SG = 1.8$ falls through water at a constant speed $U$. Determine $U$ if the cube falls (a) as oriented in Fig. P9.45a, (b) as oriented in Fig. P9.45b.

![Diagram of cube falling through water](image)

**Figure P9.50**

For steady fall, $\Sigma F = ma = 0$

or $W = \theta + F_B$, where $W =$ weight $= 500N$

$F_B =$ buoyant force $= \gamma D^3$

and $\theta = \frac{1}{2} \rho U C_D A = \text{drag}$

But,

$W = \gamma_c D^3 = SG \gamma D^3$, or $500N = 1.8 (9.80 \times 10^3 \frac{N}{m^3}) D^3$

Thus, $D = 0.305 \ m$ so that from Eq. (i)

$500N = \frac{1}{2} (999 \frac{kg}{m^3}) U^2 C_D (0.305 m)^2 + (9.80 \times 10^3 \frac{N}{m^3})(0.305 m)^3$

or $U^2 C_D = 4.78$ where $U \sim \frac{m}{s}$

(a) For case (a) $C_D = 0.80$ (see Fig. 9.29)

Hence, $U = \left( \frac{4.78}{0.80} \right)^{\frac{1}{2}} = 2.44 \frac{m}{s}$

(b) For case (b) $C_D = 1.05$

Hence, $U = \left( \frac{4.78}{1.05} \right)^{\frac{1}{2}} = 2.13 \frac{m}{s}$
A snowflake with a diameter of 0.15 in. is observed to fall through still air with a speed of 2.5 ft/s. The drag coefficient is assumed to be 1.5. (a) Estimate the weight of the snowflake.

(b) Estimate the number of these snowflakes it would take, when melted, to fill a one-gallon jug with water.

(a) If the falling speed is constant, then the weight is balanced by the drag.

\[ W = \delta = \frac{1}{2} \rho U^2 A C_D \]

\[ = \frac{1}{2} \left( 0.00237 \text{ slugs/ft}^3 \right) \left( 2.5 \text{ ft/s} \right)^2 \left( \frac{0.15 \text{ ft}}{1 \text{ ft}} \right)^2 \left( 1.5 \right) = 1.36 \times 10^{-6} \text{ slugs} \text{ ft/s}^2 \]

or

\[ W = 1.36 \times 10^{-6} \text{ lb} \]

(b) Since 1 gallon = 231 in.\(^3\) = 0.1337 ft\(^3\) it follows that the weight of 1 gallon of water is

\[ W_{\text{gal}} = V_{\text{gal}} \delta, \]

or

\[ W_{\text{gal}} = 0.1337 \text{ ft}^3 \left( 62.4 \text{ lb/ft}^3 \right) = 8.34 \text{ lb} \]

Thus if \( n \) = number of snowflakes that it takes to make a gallon of water,

\[ n \cdot W_{\text{flake}} = W_{\text{gal}} \]

or

\[ n = \frac{W_{\text{gal}}}{W_{\text{flake}}} = \frac{8.34 \text{ lb}}{1.36 \times 10^{-6} \text{ lb}} = 6.13 \times 10^6 \]

That is, 6.13 million snowflakes per gallon!
9.47  A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.6.) List any assumptions used in your calculations.

For equilibrium, $\Sigma M = 0$ or

$$M_B = 2.5 \text{ ft} \cdot \Delta P + (5 + \frac{1}{2}) \text{ ft} \cdot \Delta S,$$

where

$\Delta P$ = drag on the pole and $\Delta S$ = drag on the sign

From Fig. 9.28 with $l/D < 0.1$ for the sign, $C_{DS} = 1.9$

From Fig. 9.19 if the post acts as a square rod with sharp corners $C_{DP} = 2.2$. Thus, with $V = 30 \text{ mph} = 44 \frac{ft}{s}$,

$$\Delta S = \frac{1}{2} \rho \frac{V^2}{C_{DS}} A_S = \frac{1}{2} (0.00238 \frac{slugs}{ft^3})(44 \frac{ft}{s})^2 (1.9) \left( \frac{2.2 (3.4) \text{ ft}^2}{144} \right) = 22.7 \text{ lb}$$

and

$$\Delta P = \frac{1}{2} \rho \frac{V^2}{C_{DP}} A_P = \frac{1}{2} (0.00238 \frac{slugs}{ft^3})(44 \frac{ft}{s})^2 (2.2) \left( \frac{3}{12} (5) \text{ ft}^2 \right) = 6.34 \text{ lb}$$

Thus, from Eq. (1):

$$M_B = 2.5 \text{ ft} (6.34 \text{ lb}) + (5 + \frac{1}{2}) \text{ ft} (22.7 \text{ lb}) = 162 \text{ ft-lb}$$
If for a given vehicle it takes 20 hp to overcome aerodynamic drag while being driven at 65 mph, estimate the horsepower required at 75 mph.

\[ P = \text{power} = U \Delta P = UC_d \frac{1}{2} \rho U^2 A = C_d \frac{1}{2} \rho U^3 A \]

Thus,

\[ \frac{P_{75}}{P_{65}} = \frac{C_d \frac{1}{2} \rho (65)^3 A}{C_d \frac{1}{2} \rho (55)^3 A} = \left( \frac{75}{65} \right)^3 = 1.54, \quad \text{provided the values of } C_d \text{ are independent of } U \quad (i.e., \text{Re}). \]

Hence,

\[ P_{75} = 1.54 \times P_6 = 1.54 \times (20 \text{ hp}) = 30.8 \text{ hp} \]
9.49  Repeat Problem 9.48 if a 2-m by 2.5-m flag is attached to the top of the pole. See Fig. 9.30 for drag coefficient data for flags.

For equilibrium, \( M = \frac{l_1}{2} \delta_1 + (l_2 - \frac{D_2}{2}) \delta_2 \) \hspace{1cm} (1)

where \( l_1 = 2.0 \text{ m}, \ l_2 = 2.5 \text{ m}, \) and \( D_2 = 2 \text{ m}. \)

From the solution to Problem 9.48, \( \frac{h_1}{2} \delta_1 = 7,080 \text{ N.m} \) \hspace{1cm} (2)

Also,
\[ \delta_2 = C_D \frac{1}{2} \rho U^2 l_2 D_2 \]

we obtain \( C_D = 0.08. \)

Thus,
\[ \delta_2 = 0.08 \left( \frac{1}{2} \right) (1.23 \frac{h_2}{h_2}) (20 \text{ m})^2 (2.5 \text{ m}) (2 \text{ m}) = 98.4 \text{ N} \] \hspace{1cm} (3)

By combining Eqs. (1), (2), and (3) we obtain
\[ M = 7,080 \text{ N.m} + (20 \text{ m} - 1 \text{ m}) (98.4 \text{ N}) = 8,950 \text{ N.m} \]
9.48 Determine the moment needed at the base of a 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.

For equilibrium, \( M = \frac{F}{2} \ell \) where

\[ D = C_D \frac{1}{2} \rho U^2 \ell D \]

Since \( Re = \frac{UL}{v} = \frac{(20 \text{ m})(0.12 \text{ m})}{1.46 \times 10^{-5} \text{ m}^2 / \text{s}} = 1.64 \times 10^5 \), it follows from Fig. 9.21 that \( C_D = 1.2 \).

Thus, \( D = 1.2 \left( \frac{1}{2} \right) (1.23 \text{ kg/m}^3) (20 \text{ m})^2 (20 \text{ m})(0.12 \text{ m}) = 708 \text{ N} \)

Hence, from Eq. (1)

\[ M = \frac{20m}{2} (708 \text{ N}) = 7,080 \text{ Nm} \]
9.52 How much more power is required to peddle a bicycle at 15 mph into a 20-mph headwind than at 15 mph through still air? Assume a frontal area of 3.9 ft² and a drag coefficient of \( C_d = 0.88 \).

\[ P = \text{power} = U_b^2 A \quad \text{and} \quad F_d = C_d \frac{1}{2} \rho U^2 A, \]

where \( U_b = \text{speed of the bike} \) and \( U = \text{wind speed relative to bike} \).

Thus,

\[ P = \left(22 \frac{\text{ft}}{\text{s}}\right)(0.88)(\frac{1}{2})(0.00236 \frac{\text{slug}}{\text{ft} \cdot \text{s}^2}) V^2 (3.9 \text{ ft}^2) = 0.0898 V^2 \frac{\text{ft} \cdot \text{lbf}}{\text{sec}} \]

with \( V = \frac{\text{ft}}{\text{s}} \).

\[ (i) \]

a) With a 20 mph headwind, \( V = (15 + 20) \frac{\text{mi}}{\text{hr}} (\frac{88 \text{ ft}}{60 \text{ mi/hr}}) = 51.3 \frac{\text{ft}}{\text{s}} \)

Thus,

\[ P_a = 0.0898 (51.3)^2 = 236 \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \]

b) With still air, \( V = 15 \text{ mph} = 22 \frac{\text{ft}}{\text{s}} \)

Thus,

\[ P_b = 0.0898 (22)^2 = 43.5 \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \]

Hence, need an additional power of \( P_a - P_b = (236 - 43.5) \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \left( \frac{1 \text{ hp}}{550 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}} \right) = 0.350 \text{ hp} \)
9.53 Estimate the wind velocity necessary to knock over a 10-lb garbage can that is 3 ft tall and 2 ft in diameter. List your assumptions.

If the can is about to tip around corner O, then \( \Sigma M_{O} = 0 \), or \( 1.5 \Phi = W \)

or \( 1.5 C_{D} \frac{1}{2} \rho U^2 A = W \) A typical value of \( C_{D} \) for a cylinder is \( C_{D} = 1 \) (see Fig. 9.21)

Thus,

\[
(1.5 \text{ ft})(1\text{(ft)})(1)(0.0023 \text{ slugs ft}/\text{lb})(1015 \text{ slugs/ft}^3) U^2 (2\text{ ft})(3\text{ ft}) = 10\text{ ft-lb}, \text{ where } U = \frac{ft}{s}
\]

or \( U = 30.6 \frac{ft}{s} \)
9.54 On a day without any wind, your car consumes \( x \) gallons of gasoline when you drive at a constant speed, \( U \), from point A to point B and back to point A. Assume that you repeat the journey, driving at the same speed, on another day when there is a steady wind blowing from B to A. Would you expect your fuel consumption to be less than, equal to, or greater than \( x \) gallons for this windy round-trip? Support your answer with appropriate analysis.

**Trip with the larger power lost due to aerodynamic drag will use the most gas. Let ( ), mean "no wind" and ( )\(_2\) mean "wind".**

1. **No wind:**
   \[ P_1 = C_0 \frac{1}{2} \rho \frac{U^2}{A} \] for both A→B and B→A
   Thus, \[ P_1 = \text{power} = \frac{1}{2} \rho \frac{U^2}{A} C_0 A \]

2. **Wind (\( U_w = \text{wind speed}; \text{assume} \ U_w < U \)):**
   \[ P_2 = C_0 \frac{1}{2} \rho (U+U_w)^2 A \] for A→B
   \[ P_2 = C_0 \frac{1}{2} \rho (U-U_w)^2 A \] for B→A
   Thus,
   \[ P_2 = \frac{1}{2} \rho (U+U_w)^2 C_0 A \text{ for } A \rightarrow B \]
   \[ P_2 = \frac{1}{2} \rho (U-U_w)^2 C_0 A \text{ for } B \rightarrow A \]

   Energy used = \( Pt \), where \( t = \text{time to go from } A \rightarrow B \text{ or } B \rightarrow A \)
   Thus,
   \[ E_1 = 2 \left( \frac{1}{2} \rho \frac{U^3}{A} \right) t \quad (\text{Note: Factor of } 2 \text{ for } A \rightarrow B \text{ and } B \rightarrow A) \]
   and
   \[ E_2 = \frac{1}{2} \rho (U+U_w)^2 C_0 A t + \frac{1}{2} \rho (U-U_w)^2 C_0 A t \]
   Thus,
   \[ \frac{E_1}{E_2} = \frac{2U^3}{(U+U_w)^2 U + (U-U_w)^2 U} = \frac{2U^3}{2U^3 + 2U_w^2 U} = \frac{1}{1 + (U_w/U)^2} < 1 \]

   Hence,
   \[ \frac{E_1}{E_2} < 1, \text{ i.e. more fuel needed when windy} \]
By paying close attention to design details, the drag coefficient for a typical car has been reduced over the years as indicated in Fig. 9.27. How fast can one drive a 2005 style car if it is to have the same aerodynamic drag as a 1940 model being driven at 65 mph. Assume that the frontal area of the 2005 model is 85% that of the 1940 model.

\[ D_{40} = 0.95 \]

or

\[ \frac{1}{2} \rho U_{40}^2 A_{40} C_{D40} = \frac{1}{2} \rho U_{05}^2 A_{05} C_{D05}, \]  

where \( U_{40} = 65 \text{ mph} \), \( A_{05} = 0.85 A_{40} \) and from Fig. 9.21

\[ C_{D40} = 0.57, \ C_{D05} = 0.30 \]

Thus,

\[ U_{40}^2 A_{40} C_{D40} = U_{05}^2 A_{05} C_{D05} \]

or

\[ U_{05} = \left( \frac{A_{05} C_{D05}}{A_{40} C_{D40}} \right)^{1/2} \]

\[ U_{40} = \left( \frac{0.57}{0.85(0.30)} \right)^{1/2} \text{ (65 mph)} = 97.2 \text{ mph} \]
9.56 As shown in Video V9.8 and Fig. P9.56, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction of how many horsepower needed at a highway speed of 65 mph?

\[ P = \text{power} = \frac{1}{2} \rho \frac{D}{A} \]

Thus, \( \Delta P = \text{reduction in power} \)

\[ = P_b - P_a \]

\[ = \frac{1}{2} \rho \frac{D^3}{A} [C_{D_b} - C_{D_a}] \]

With $\nu = 65 \text{ mph} = 95.3 \text{ fps}$,

\[ \Delta P = \frac{1}{2} \left( \frac{0.00238 \text{ slugs}}{\text{ft}^3} \right) \left( 95.3 \text{ ft/s} \right)^3 (10\text{ft})(12\text{ft}) [0.96 - 0.70] \]

\[ = 32,100 \frac{\text{ft-lb}}{s} \left( \frac{1 \text{ hp}}{550 \frac{\text{ft-lb}}{s}} \right) = 58.4 \text{ hp} \]
9.57 The structure shown in Fig. P9.57 consists of a cylindrical support post to which a rectangular flat-plate sign is attached. Estimate the drag on the structure when a 50-mph wind blows against it.

\[ D = D_{\text{sign}} + D_{\text{post}}, \quad \text{where} \quad D_{\text{sign}} = \frac{1}{2} \rho U^2 A_{\text{sign}} C_{D_{\text{sign}}} \quad \text{and} \quad D_{\text{post}} = \frac{1}{2} \rho U^2 A_{\text{post}} C_{D_{\text{post}}} \]

Also, \( A_{\text{sign}} = 12 \text{ ft} \times 24 \text{ ft} = 288 \text{ ft}^2 \) and
\[ A_{\text{post}} = 3 \text{ ft} \times 30 \text{ ft} = 90 \text{ ft}^2 \]

From Fig. 9.28, for a thin flat plate with \( L/D \leq 0.1 \), \( C_D = 1.9 \)

Thus, \( C_{D_{\text{sign}}} = 1.9 \)

Also, for the cylinder (post), \( Re = \frac{UD}{\nu} \), where
\[ U = 50 \text{ mi/hr}, \quad \frac{1}{hr} = \frac{720 \text{ ft}}{60 \text{ s}} = 72 \text{ ft/s} \]
so that
\[ Re = \frac{(73.3 \text{ ft/s})(3 \text{ ft})}{1.57 \times 10^{-4} \text{ ft/lb/s}} = 1.40 \times 10^6 \]

Hence, from Fig. 9.21, \( C_{D_{\text{post}}} = 0.8 \)

By using the above data, Eq. (1) gives
\[ D = \frac{1}{2} \rho U^2 [A_{\text{sign}} C_{D_{\text{sign}}} + A_{\text{post}} C_{D_{\text{post}}}]
\]
\[ = \frac{1}{2} (0.00238 \text{ lb/ft}^2)(73.3 \text{ ft/s})^2 \times [288 \text{ ft}^2(1.9) + 90 \text{ ft}^2(0.8)] \]

or
\[ D = 3960 \text{ lb} \]
As shown in Video V9.5 and Fig. P9.59, a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) curls up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.

For equilibrium conditions

\[ W = \frac{1}{2} \rho v^2 A \]

Assume \( W = 160 \text{ lb} \) and \( C_D A = 9 \text{ ft}^2 \) (see Fig. 9.30).

Thus,

\[ 160 \text{ lb} = \left( \frac{1}{2} \right) (0.00238 \text{ slugs/ft}^3) U^2 (9 \text{ ft}^2) \text{ where } U \sim \frac{f}{s} \]

or

\[ U = \left( \frac{122 \text{ ft}}{s} \right) \left( \frac{1 \text{ mi}}{5280 \text{ ft}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 83.2 \text{ mph} \]

Note: If the skydiver "curled up into a ball", then \( C_D A = 2.5 \text{ ft} \) (see Fig. 9.30) and \( U = 158 \text{ mph} \)
For the balloon to remain stationary
\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]
Thus, \( \theta = \frac{T \cos \theta}{m} \) or \( T = \frac{F_y}{\cos \theta} \)
and
\[ F_B = W + T \sin \theta + W_{He} \]
which combine to give
\[ F_B = W + \theta \tan \theta + W_{He} \]
But \( W = 0.2 \text{ lb} \), \( F_B = \rho g V = \left( \frac{7.65 \times 10^{-2}}{6.2} \right) \frac{2 \pi}{2} \left( \frac{2}{3} \right)^3 = 0.3204 \text{ lb} \)
and \( W_{He} = \rho_{He} V = \left( 0.011 \frac{100}{6.2} \right) \frac{2 \pi}{2} \left( \frac{2}{3} \right)^3 = 0.0461 \text{ lb} \)
Thus, Eq. (1) becomes
\[ 0.3204 \text{ lb} = 0.2 \text{ lb} + \theta \tan \theta + 0.0461 \text{ lb} \]
or
\[ \theta \tan \theta = 0.0743 \text{ lb} \]
Also, \( \theta = \frac{C_D}{2} \frac{U^2}{2} \frac{D^2}{2} \)
\[ = C_D U^2 (0.00239 \frac{\text{ft}}{\text{s}}) \left( \frac{2 \text{ ft}}{2} \right)^2 \]
\[ = 0.00374 C_D U^2 \text{ lb}, \text{ where } U \approx \frac{5}{3} \]

Hence,
\[ 0.00374 C_D U^2 \tan \theta = 0.0743 \quad \text{or} \quad \tan \theta = \frac{1.95}{C_D U^2} \quad \text{(2)} \]
Also, \( Re = \frac{\rho U}{\nu} \) or \( Re = 1.27 \times 10^4 U \quad \text{(3)} \)
and from Fig. 9.21:
\[ C_D = \frac{D}{C_D} \quad \text{(4)} \]

Thus, select various \( 1 \text{ mph} \leq U \leq 50 \text{ mph} \) (i.e., \( 1.47 \frac{\text{ft}}{\text{s}} \leq U \leq 73.3 \frac{\text{ft}}{\text{s}} \)) and use Eqs. (2), (3), (4) to obtain \( \theta \). Plotted results are shown below.
Note: Because of the sudden change in $C_D$ when the boundary layer becomes turbulent (at about 15 mph), the $\Theta$ vs $U$ curve is highly non-linear. In fact, for some values of $\Theta$ there is more than one possible value of $U$. It would not work well as a wind speed indicator in this range.
9.61 A 0.30-m-diameter cork ball (SG = 0.21) is tied to an object on the bottom of a river as is shown in Fig. P9.61. Estimate the speed of the river current. Neglect the weight of the cable and the drag on it.

For the ball to remain stationary
\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]
Thus, \[ \theta = T \cos 30^\circ \quad \text{or} \quad T = \frac{\theta}{\cos 30^\circ} \]
and
\[ F_B = W + T \sin 30^\circ \]
Hence, \[ F_B = W + \theta \tan 30^\circ \]
where \[ F_B = \rho g V = \left(2.80 \frac{kN}{m^3}\right) \left(\frac{4}{3} \left(\frac{0.30}{2}\right)^3\right) \]
\[ = 0.1385 \ kN \]
and
\[ W = \chi_c \gamma = \left(\frac{\chi_c}{\gamma}\right) \gamma = (SG)F_B \]
\[ = 0.21(0.1385 \ kN) \]
\[ = 0.0291 \ kN \]
Thus, \[ 0.1385 \ kN = 0.0291 \ kN + \theta \tan 30^\circ \]
or \[ \theta = 0.189 \ kN, \quad \text{where} \quad \theta = C_D \frac{1}{2} \rho U^2 A = \frac{C_D}{0.30^2}(\frac{999 \ kg}{m^3})(\frac{U}{0.30^2})^2 \]
\[ = 35.3 \ C_D U^2 \ N, \quad \text{where} \quad U = \frac{m}{s} \]
Hence
\[ 35.3 C_D U^2 \times 189 \]
\[ \text{or} \quad C_D U^2 = 5.35 \]
Also, \[ Re = \frac{U D}{\nu} = \frac{(0.30)U}{1.12 \times 10^{-6} m^2/s} = 2.68 \times 10^5 U \]
and
from Fig. 9.21
\[ C_D \]
\[ \text{Re} \]
\[ \text{Trial and error solution for } U: \text{Assume } C_D; \text{ calculate } U \text{ from Eq. (1)} \]
and \[ Re \text{ from Eq. (2)}; \text{ check } C_D \text{ from Eq. (3), the graph.} \]
Assume \[ C_D = 0.5 \rightarrow U = 3.27 \frac{m}{s} \rightarrow Re = 8.76 \times 10^5 \rightarrow C_D = 0.15 \pm 0.5 \]
Assume \[ C_D = 0.15 \rightarrow U = 5.97 \frac{m}{s} \rightarrow Re = 1.60 \times 10^6 \rightarrow C_D = 0.20 \pm 0.15 \]
Assume \[ C_D = 0.19 \rightarrow U = 5.31 \frac{m}{s} \rightarrow Re = 1.42 \times 10^6 \rightarrow C_D = 0.19 \text{ (checks)} \]
Thus, \[ U = 5.31 \frac{m}{s} \]

9-58
9.62 Air flows past two equal sized spheres (one rough, one smooth) that are attached to the arm of a balance as is indicated in Fig. P9.62. With $U = 0$ the beam is balanced. What is the minimum air velocity for which the balance arm will rotate clockwise?

For clockwise rotation to start, $\Sigma M_0 < 0$

That is $0.3 \Delta t = 0.5 \Delta t_2$, where

$\Delta t = C_{D_1} \frac{1}{2} \rho U^2 \frac{A_1}{A}$ and

$\Delta t_2 = C_{D_2} \frac{1}{2} \rho U^2 \frac{A_2}{A}$

Thus, $0.3 C_{D_1} \rho U^2 \frac{A_1}{A} = 0.5 C_{D_2} \rho U^2 \frac{A_2}{A}$, or since $U_1 = U_2$ and $A_1 = A_2$ this gives $C_{D_2} = 0.6 C_{D_1}$

Consider the curves in Fig. 9.25 with $\frac{D}{D_1} = 0$ and $\frac{D}{D_2} = 1.25 \times 10^{-2}$

Trial and error solution to find $Re$ so that Eq.(1) is satisfied.

Assume $Re = 6 \times 10^4 \rightarrow C_{D_1} = 0.5, C_{D_2} = 0.46$ or $C_{D_2} = 0.92 \times 0.6$

Assume $Re = 8 \times 10^4 \rightarrow C_{D_1} = 0.5, C_{D_2} = 0.21$ or $C_{D_2} = 0.42 \times 0.6$

Assume $Re = 7 \times 10^4 \rightarrow C_{D_1} = 0.5, C_{D_2} = 0.33$ or $C_{D_2} = 0.66 \times 0.6$

Thus, $Re \approx 7.1 \times 10^4 = \frac{UD}{\nu} = \frac{0.1 m \times U}{1.44 \times 10^{-8} \text{ m}^2}$ or $U = 10.4 \text{ m s}^{-1}$

9-60
9.63 A radio antenna on a car consists of a circular cylinder \( \frac{1}{4} \) in. in diameter and 4 ft long. Determine the bending moment at the base of the antenna if the car is driven 55 mph through still air.

For equilibrium, \( \Sigma M_0 = 0 \), or \( M_0 = \frac{1}{2} \tau A \)

where \( \tau = C_D \frac{1}{2} p U^2 A \)

Since \( Re = \frac{U D}{v} = \frac{(80.7 \text{ ft}) (\frac{1}{4} \text{ ft})}{1.57 \times 10^{-5} \text{ ft}^2/\text{s}} = 1.07 \times 10^4 \),

it follows from Fig. 9.21 that \( C_D = 1.3 \)

Hence \( \tau = 1.3 \left( \frac{1}{2} \right) (0.00238 \text{ slugs ft/s}) (80.7 \text{ ft}) (\frac{1}{4} \text{ ft}) (4 \text{ ft}) (\frac{1}{16} \text{ ft}) = 0.840 \text{ lb} \)

Thus, \( M_0 = (2 \text{ ft})(0.840 \text{ lb}) = 1.68 \text{ ft} \cdot \text{lb} \)
9.64 Determine the power needed to overcome the aerodynamic drag of the antenna in Problem 9.63.

\[ \mathcal{P} = \text{power} = U\mathcal{F} = U[C_D \frac{1}{2} \rho U^2 A] \]

or

\[ \mathcal{P} = C_D \frac{1}{2} \rho U^3 A, \quad \text{where} \quad U = 55 \text{ mph} = 80.7 \frac{ft}{s} \]

Since \( \text{Re} = \frac{U D}{V} = \frac{(80.7 \frac{ft}{s})(\frac{1}{2} \frac{ft}{s})(80.7 \frac{ft}{s})}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 1.07 \times 10^4 \), it follows from Fig. 9.21 that \( C_D = 1.3 \).

Hence,

\[ \mathcal{P} = 1.3 \left( \frac{1}{4} \right) \left( \frac{0.00238 \frac{ft}{s}}{\frac{ft}{s}} \right)(80.7 \frac{ft}{s})^3 \left( \frac{1}{2} \frac{ft}{s} \right)(\frac{1}{2} \frac{ft}{s}) = 67.8 \frac{ft \cdot lb}{s} \left( \frac{1 \text{ hp}}{550 \frac{ft \cdot lb}{s}} \right) \]

\[ = 0.123 \text{ hp} \]
9.65 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

\[ \mathcal{D} = C_D \frac{1}{2} \rho U^2 A \], where \( U = (55 \text{ mph}) \left( \frac{88 \frac{ft}{hr}}{60 \text{ mph}} \right) = 80.7 \frac{ft}{hr} \)

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with \( C_D = 1.1 \) (see Fig. 9.29).

Thus, \( \mathcal{D} = (1.1) \left( \frac{1}{2} \right) (0.00236) \left( 80.7 \frac{ft}{hr} \right)^2 \left( \frac{4}{12} \text{ ft} \right) \left( \frac{6}{12} \text{ ft} \right) = 1.42 \text{ lb} \)

If your hand is normal to the the lift force is zero.

For \( U = 550 \text{ mph} = 807 \frac{ft}{hr} \) (i.e., a 10 fold increase in \( U \)) the drag will increase by a factor of 100 (i.e., \( \mathcal{D} \propto U^2 \)), or \( \mathcal{D} = 142 \text{ lb} \)

Note: We have assumed that \( C_D \) is not a function of \( U \). That is, it is not a function of either \( Re = \frac{U D}{\nu} \) or \( Ma = \frac{U}{c} \).
A 2-mm-diameter meteor of specific gravity 2.9 has a speed of 6 km/s at an altitude of 50,000 m where the air density is $1.03 \times 10^{-3}$ kg/m³. If the drag coefficient at this large Mach number condition is 1.5, determine the deceleration of the meteor.

$$D = ma \quad \text{where} \quad m = \rho \frac{4}{3} \pi \left(\frac{D}{2}\right)^3 = (2.9) \left(\frac{199.5 \text{ kg}}{m^3}\right) \frac{4}{3} \pi \left(\frac{2 \times 10^{-3} \text{ m}}{2}\right)^3$$

$$= 1.21 \times 10^{-6} \text{ kg}$$

Also, $$D = C_d \frac{1}{2} \rho U^2 A$$

$$= 1.5 \left(\frac{1}{2}\right) \left(1.03 \times 10^{-3} \text{ kg/m}^3\right) \left(6 \times 10^3 \text{ m/s}\right)^2 \frac{4}{3} \pi \left(2 \times 10^{-3} \text{ m}\right)^2 = 8.74 \times 10^{-2} \text{ N}$$

Thus, $$a = \frac{D}{m} = \frac{8.74 \times 10^{-2} \text{ N}}{1.21 \times 10^{-6} \text{ kg}} = 7220 \text{ m/s}^2$$
9.68 A 30-ft-tall tower is constructed of equal 1-ft segments as is indicated in Fig. P9.68. Each of the four sides is similar. Estimate the drag on the tower when a 75-mph wind blows against it.

Assume no interference between the front and back portions of the tower. Also, neglect the drag on the sides of the tower. Hence, for thirty one-foot segments

\[ \delta = 30 \left( \frac{1}{2} \rho U^2 \right) \left[ (C_{D1} A_1 + C_{D2} A_2)_{\text{front}} + (C_{D1} A_1 + C_{D2} A_2)_{\text{back}} \right] \]

where

\[ U = 75 \text{ mph} \left( \frac{88 \text{ ft}}{60 \text{ mph}} \right) = 110 \frac{\text{ft}}{\text{s}} \]

From Fig. 9.23, \( C_{D1} = 1.98 \) \( \uparrow \) \( \downarrow \), \( C_{D2} = 1.82 \)

Thus, from Eq. (1)

\[ \delta = 30 \left( \frac{1}{2} \cdot 0.0023 \cdot \frac{\text{slug}}{\text{ft}^3} \cdot 110^2 \frac{\text{ft}^2}{\text{s}^2} \right) \left[ (1.98)(2)(1 \text{ ft})(\frac{1}{2} \text{ ft}) + (1.9)\left(\frac{1}{2} \text{ ft}\right)\left(\frac{8+8+8\sqrt{2}}{12} \text{ ft} \right) \right] \\
+ (1.82)(2)(1 \text{ ft})(\frac{1}{2} \text{ ft}) + (1.7)\left(\frac{1}{2} \text{ ft}\right)\left(\frac{8+8+8\sqrt{2}}{12} \text{ ft} \right) \]

or

\[ \delta = 859 \text{ lb} \]
A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.69 and Video V3.1. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.

For equilibrium, \( \dot{Q} = W \) or \( C_D \frac{1}{2} \rho V_2^2 A = W \), where \( A = \frac{\pi D^2}{4} \)

Thus,

\[
V_2 = \left[ \frac{2W}{C_D \rho \pi D^2/4} \right]^{1/2} = \left[ \frac{8 \text{ (0.14 lb)}}{0.5 \text{ (0.00238 \frac{slugs}{ft^3})} \pi \left( \frac{2}{72 \frac{ft}{s}} \right)^2} \right]^{1/2} = 104 \frac{ft}{s}
\]

Also,

\[ V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_1 = V_2 \frac{A_2}{A_1} = (104 \frac{ft}{s}) \frac{0.3 \frac{ft^2}{s}}{0.6 \frac{ft^2}{s}} = 52.0 \frac{ft}{s} \]

and

\[ \rho_1 \frac{1}{2} \rho V_1^2 = \rho_2 + \frac{1}{2} \rho V_2^2 \quad \text{where} \quad \rho_2 = 0 \]

Thus,

\[ \rho_1 \frac{1}{2} \rho [V_2^2 - V_1^2] = \frac{1}{2} \left( 0.00238 \frac{\text{slugs}}{\text{ft}^3} \right) \left( 104 \frac{ft}{s} \right)^2 - (52.0 \frac{ft}{s})^2 \]

\[ = 9.65 \frac{lb}{\text{ft}^2} \]
The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. (a) Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. (b) Repeat your calculation if the velocity profile against the building is a typical profile for an urban area (see Problem 9.17) and the wind speed halfway up the building is 20 m/s.

\[ F = C_D \frac{1}{2} \rho U^2 A = 1.3 \left( \frac{1}{2} \right) (1.23 \frac{kg}{m^3}) (20 \frac{m}{s})^2 (154m)(875m) \]

or

\[ F = 4.31 \times 10^6 \text{ N} = 4.31 \text{ MN} \]

(b) For an urban area, \( U = C \cdot y^{0.4} \).

Thus, with \( U = 20 \frac{m}{s} \) at \( y = \frac{h}{2} = 77m \)

we obtain

\[ C = \frac{20}{77} \approx 3.52 \]

, or \( U = 3.52 \cdot y^{0.4} \) with \( U \sim \frac{g}{y}, y \sim m \)

The total drag is

\[ F = \int dF = \int C_D \frac{1}{2} \rho U^2 dA = \frac{1}{2} \rho C_D \int (3.52 \cdot y^{0.4})^2 (875) dy \]

or

\[ F = \frac{1}{2} (1.23)(1.3)(3.52)^2(875) \int_0^{154} y^{0.8} dy = 3.67 \left( \frac{1}{1.5} \right)(154)^{1.8} = 4.17 \times 10^6 \text{ N} \]

Thus,

\[ F = 4.17 \text{ MN} \]
9.71 A 0.3-in.-diameter wire is draped across a river in which the water velocity is 3 ft/s. The length of the wire in the water is 420 ft. Determine the horizontal and vertical components of force of the water on the wire.

The horizontal force is equal to the drag, \( D \):

\[ D = C_D \frac{1}{2} \rho U^2 A \]

where \( C_D \) is a function of the Reynolds number.

Thus, with

\[ Re = \frac{\rho U D}{\mu} = \frac{1.94 \text{ slugs/ft}^3 (3 \text{ ft/s}) (0.3 \text{ ft}^2)}{2.34 \times 10^{-5} \text{ lb s/ft}^2} = 6.22 \times 10^3 \]

it follows from Fig. 9.21 that

\( C_D = 1.3 \)

Hence,

\[ D = 1.3 \left( \frac{1}{2} \right) (1.94 \text{ slugs/ft}^3) (3 \text{ ft/s})^2 (0.3 \text{ ft}) (420 \text{ ft}) = 119 \text{ lb} \]

The vertical force is equal to the buoyant force, \( F_B \):

\[ F_B = \delta^2 V = 62.4 \frac{\text{lb}}{ \text{ft}^3} (420 \text{ ft}) \left( \frac{0.3}{72} \text{ ft} \right)^2 = 12.9 \text{ lb} \]
9.72 The fishing lure shown in Fig. P9.72 floats on the surface of the water when there is no tension, T, in the line. However, it is designed to sink when reeled in. If it is reeled in too fast (i.e., \( U \) is too large) it will sink to the bottom and get snagged. Show that the depth (or equivalently, \( \theta \)) increases as \( U \) increases.

Start with a free-body diagram of the lure including the tension in the line, the weight of the lure, the buoyant force, the lift

\[ T = \text{tension in line} \]
\[ F_B = \text{buoyant force} \]
\[ \vec{D} = \text{drag} \]
\[ \vec{W} = \text{weight} \]
\[ \vec{L} = \text{lift} \]

A free body diagram of the lure being pulled through the water is shown below. Note that the lift is shown acting downward, line an inverted wing or a spoiler on a race car.

Thus, \( \sum F_x = 0 \) and \( \sum F_y = 0 \) give

\[ T \cos \theta = \vec{D} \] \hspace{1cm} (1)
and
\[ T \sin \theta = \vec{L} + \vec{W} - F_B \] \hspace{1cm} (2)

Dividing Eq. (1) by Eq. (2) gives

\[ \tan \theta = \frac{\vec{L} + \vec{W} - F_B}{T \sin \theta} = \frac{C_B \frac{1}{2} \rho U^2 A + \vec{W} - F_B}{C_D \frac{1}{2} \rho U^2 A} = \frac{C_B + \frac{\vec{W} - F_B}{(\rho \vec{U}^2 A)}}{C_D} \]

or

\[ \tan \theta = a - \frac{b}{U^2} \]

where \( a = \frac{C_B}{C_D} \) and \( b = \frac{2(F_B - \vec{W})}{C_D \rho A} \) are positive constants

(Note that \( F_B \) > \( \vec{W} \) because the lure floats on the surface when \( T = 0 \))

From Eq. (3) the graph of \( \tan \theta \) vs \( U \) looks as shown. Thus, as \( U \) increases \( \tan \theta \) increases. That is, as \( U \) increases \( \theta \) increases.
9.73 A regulation football is 6.78 in. in diameter and weighs 0.91 lb. If its drag coefficient is \( C_D = 0.2 \), determine its deceleration if it has a speed of 20 ft/s at the top of its trajectory.

\[
\begin{aligned}
\frac{D}{g} &= ma \quad \text{where} \quad m = \frac{W}{g} = \frac{0.91 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.0283 \text{ slugs} \\
\text{and} \\
\frac{D}{2} &= C_D \frac{1}{2} \rho U^2 A = 0.2 \left( \frac{1}{2} \right) (0.00238 \text{ slugs ft}^3) (20 \text{ ft/s})^2 \left( \frac{\pi}{4} \left( \frac{6.78 \text{ ft}}{12} \right)^2 \right) = 0.0239 \text{ lb} \\
\text{Thus,} \\
\frac{D}{m} &= \frac{0.0239 \text{ lb}}{0.0283 \text{ slugs}} = 0.841 \text{ ft/s}^2
\end{aligned}
\]
9.75 The paint stirrer shown in Fig. P9.75 consists of two circular disks attached to the end of a thin rod that rotates at 80 rpm. The specific gravity of the paint is $SG = 1.1$ and its viscosity is $\mu = 2 \times 10^{-3} \text{lb} \cdot \text{s}/\text{ft}^2$. Estimate the power required to drive the mixer if the induced motion of the liquid is neglected.

If we neglect the effects of the shaft and rod and consider the paint to be stationary, then

$$M = 2 \omega R$$

where $M =$ torque to rotate shaft

and $\omega = \text{d} \theta / \text{d} t$ on one disk = $C_D \frac{1}{2} \rho U^2 A$

Also, $U = \omega R$ and $P =$ power to rotate shaft = $M \omega$

Thus, $P = 2 \omega R \omega = 2 C_D \frac{1}{2} \rho (\omega R)^2 \frac{1}{2} \rho \omega R \omega$

or

$$P = \frac{\rho}{\mu} C_D \rho \omega^2 R^3 \frac{1}{2} = \frac{\rho}{\mu} C_D \rho U^3 \frac{1}{2}$$

where $P = SG \rho h_2 \omega$

$$Re = \frac{\rho UD}{\mu}$$

where

$$U = \omega R = \left( 80 \frac{\text{rev}}{\min} \right) \left( \frac{1}{60} \frac{\text{min}}{\text{rev}} \right) \left( \frac{2\pi}{12} \frac{\text{rad}}{\text{rev}} \right) = \frac{1.57}{12} \frac{\text{ft}}{s}$$

we have

$$Re = \frac{1}{(1.1)(1.94 \frac{\text{ft}}{s})(1.353 \frac{\text{ft}}{s})} \left( \frac{7}{8(12)} \frac{\text{ft}}{s} \right) = 10.5$$

For a circular disk, $C_D = 1.1$ if $Re > 10^3$ (see Fig. 9.24) while $C_D = \frac{20 \sqrt{Re}}{Re}$ if $Re < 10^3$ (see Table 9.4)

For this particular problem $1 < Re = 10.5 < 10^3$

Note: If the low Reynolds number result (Eq. 20) is valid up to $Re = 10^5$,

then $C_D = \frac{20 \sqrt{Re}}{Re} = 1.94$

To be on the conservative side (i.e., maximum power), use the larger $C_D = 1.94$ from Eq. (1)

$$P = \frac{\pi}{4} (1.94)(1.1)(1.94 \frac{\text{ft}}{s})(1.353 \frac{\text{ft}}{s})^3 \left( \frac{7}{8(12)} \frac{\text{ft}}{s} \right)^3$$

$$= 0.0428 \frac{\text{ft}^2 \cdot \text{lbf}}{\text{s}}$$

or

$$P = \left( \frac{0.0428 \frac{\text{ft}^2 \cdot \text{lbf}}{\text{s}}}{550 \frac{\text{ft}^2 \cdot \text{lbf}}{\text{s}}} \right) \left( \frac{1 \text{ hp}}{550 \frac{\text{ft}^2 \cdot \text{lbf}}{\text{s}}} \right) = 7.78 \times 10^{-5} \text{ hp}$$
9.77 The football shown in Fig. P9.77 can be blown from its kicking tee by pivoting freely about point (1). The drag coefficient (based on the cross-sectional area of the football, \(\pi D^2/4\)) is a function of its orientation, \(\theta\), as shown. Plot a graph of the wind speed needed to blow the football from the tee as a function of the angle \(\theta\).

![Diagram of football and wind](image)

**FIGURE P9.77**

From the free body diagram of the football, summing moments about the pivot point (1) gives

\[ \sum M_1 = 0 \quad \text{or} \quad W \ell \cos \theta = D \ell \sin \theta \]

Thus, \(W = \ell \tan \theta = C_D \frac{1}{2} \rho \ell^2 A \tan \theta\) which gives

\[ U = \left[ \frac{2W}{C_D \rho A \tan \theta} \right]^{1/2} \]

or

\[ U = \left[ \frac{2(0.9116)}{C_D(0.00238 \text{ slugs/ft}^3) \frac{\pi}{4}(6.8 \text{ in})^2 \tan \theta} \right]^{1/2} \]

Hence, \(U = 55.1 \sqrt{1/(C_D \tan \theta)}\)

Using values of \(C_D\) from Fig. P9.77, the following results are obtained:

<table>
<thead>
<tr>
<th>(\theta, \text{deg})</th>
<th>drag coef, CD</th>
<th>(U, \text{ft/s})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.26</td>
<td>208.8</td>
</tr>
<tr>
<td>30</td>
<td>0.36</td>
<td>120.9</td>
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<td>0.60</td>
<td>71.1</td>
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<tr>
<td>60</td>
<td>1.00</td>
<td>41.9</td>
</tr>
<tr>
<td>75</td>
<td>1.17</td>
<td>26.4</td>
</tr>
<tr>
<td>90</td>
<td>1.20</td>
<td>0</td>
</tr>
</tbody>
</table>
An airplane tows a banner that is \( b = 0.8 \text{ m} \) tall and \( l = 25 \text{ m} \) long at a speed of 150 km/hr. If the drag coefficient based on the area \( bl \) is \( C_D = 0.06 \), estimate the power required to tow the banner. Compare the drag force on the banner with that on a rigid flat plate of the same size. Which has the larger drag force and why?

\[
\mathcal{P} = D \bar{U}, \quad \text{where} \quad D = C_D \frac{1}{2} \rho \bar{V}^2 A \quad \text{with} \quad A = bl.
\]

Thus, with \( C_D = 0.06 \) and \( \bar{V} = (150 \text{ km/hr}) \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) (\frac{1000 \text{ m}}{1 \text{ km}}) = 41.7 \text{ m/s} \), this gives

\[
\mathcal{P} = (0.06) (\frac{1}{2}) (1.23 \frac{\text{ kg}}{\text{ m}^3}) (41.7 \frac{\text{ m}}{\text{ s}})^3 (0.8 \text{ m})(25 \text{ m}) = 5.35 \times 10^3 \text{ W} = 5.35 \text{ kW}
\]

For a rigid flat plate

\[
\mathcal{P} = D \bar{U} = 2 C_D \frac{1}{2} \rho \bar{V}^3 bl \quad \text{(the factor of two is needed because the drag coefficient is based on the drag on one side of the plate)}
\]

With \( R_e = \frac{U_l}{V} = \frac{(41.7 \text{ m/s})(25 \text{ m})}{1.44 \times 10^5 \frac{\text{ m}}{\text{ s}}} = 7.14 \times 10^7 \), we obtain from Fig. 9.15 a value of \( C_D \approx 0.0025 \) for a smooth plate.

Thus,

\[
\mathcal{P} = 2 (0.0025) (\frac{1}{2}) (1.23 \frac{\text{ kg}}{\text{ m}^3}) (41.7 \frac{\text{ m}}{\text{ s}})^3 (0.8 \text{ m})(25 \text{ m}) = 4.46 \times 10^3 \text{ W} = 4.46 \text{ kW}
\]

For the flat plate case, the drag is relatively small because it is due entirely to shear (viscous) forces. Due to the "fluttering" of the banner, a good portion of its drag (and hence power) is a result of pressure forces. It is not as streamlined as a rigid flat plate.
9.79 By appropriate streamlining, the drag coefficient for an airplane is reduced by 18% while the frontal area remains the same. For the same power output, by what percentage is the flight speed increased?

\[ P = \frac{1}{2} \rho U^2 A, \quad \text{where} \quad A = C_D \frac{1}{2} \rho U^2 A \]

Let \((A)_{0}\) denote the original configuration and \((A)_{s}\) the streamlined one. Thus, with \(P_{0} = P_{s}\) we obtain

\[ C_{D_{0}} \frac{1}{2} \rho_{0} U_{0}^2 A_{0} U_{0} = C_{D_{s}} \frac{1}{2} \rho_{s} U_{s}^2 A_{s} U_{s} \quad \text{or with} \quad A_{0} = A_{s} \quad \rho_{0} = \rho_{s} \]

\[ \frac{U_{0}^2}{C_{D_{0}}} = \frac{U_{s}^2}{C_{D_{s}}} \quad \text{Thus,} \quad \frac{U_{s}}{U_{0}} = \frac{\sqrt[3]{C_{D_{0}}}}{\sqrt[3]{C_{D_{s}}}} = \frac{C_{D_{0}}}{\left[C_{D_{0}} - 0.18C_{D_{0}}\right]^{\frac{1}{3}}} = 1.068 \]

i.e., a 6.84% speed increase

Note: \(P \sim U^3 C_D \), so that \(\delta P = 3U^2 C_D \delta U + U^3 \delta C_D \). Thus, with \(\delta P = 0\), this gives

\[ \frac{\delta U}{U} = -\frac{1}{3} \frac{\delta C_D}{C_D} = -\frac{-0.18}{3} = +0.06 = 6\% \]
9.80 The blimp shown in Fig. P9.80 is used at various athletic events. It is 128 ft long and has a maximum diameter of 33 ft. If its drag coefficient (based on the frontal area) is 0.060, estimate the power required to propel it (a) at its 35-mph cruising speed, or (b) at its maximum 55-mph speed.

\[ P = \rho U^2 A \]

where \( \rho = C_d \frac{1}{2} \rho V^2 A \)

Thus, with

\[ \rho = 0.060 \left( \frac{1}{2} \right) \left( 0.00238 \frac{\text{slug}}{\text{ft}^2} \right) \frac{\pi}{4} (33 \text{ ft})^2 \]

\[ = 0.0611 \text{ slug/ft}^2 \text{ lb}, \text{ where } U = \text{ ft/s} \]

(a) Thus, with \( U = 35 \frac{\text{mi}}{\text{hr}} \left( \frac{5280 \text{ ft/mi}}{3600 \text{ s/hr}} \right) = 51.3 \text{ ft/s}, \)

\[ \rho = 0.0611 (51.3)^2 = 161 \text{ lb} \]

so that

\[ P = \rho U = 161 \text{ lb} (51.3 \text{ ft/s}) \left( \frac{1 \text{ hp}}{550 \text{ ft-lb/s}} \right) = 15.0 \text{ hp} \]

(b) Similarly, with \( U = 55 \text{ mph} = 80.7 \text{ ft/s}, \)

\[ \rho = 0.0611 (80.7)^2 = 398 \text{ lb} \]

so that

\[ P = \rho U = 398 \text{ lb} (80.7 \text{ ft/s}) \left( \frac{1 \text{ hp}}{550 \text{ ft-lb/s}} \right) = 58.4 \text{ hp} \]
Estimate the power needed to overcome the aerodynamic drag of a person who runs at a rate of 100 yds in 10 s in still air. Repeat the calculations if the race is run into a 20-mph headwind; a 20 mph tailwind. Explain.

In still air, \( P = \delta U \), where \( \delta = C_d \frac{1}{2} \rho U^2 A \) with \( U = \frac{300 \text{ ft}}{10 \text{ s}} = 30 \text{ ft/s} \).

From Fig. 9.30, \( C_d A = 9 \text{ ft}^2 \)

Thus, \( \delta = \left( \frac{1}{2} \right) (0.00238 \frac{\text{slugs}}{\text{ft}^2}) (30 \text{ ft/s})^2 (9 \text{ ft}^2) = 9.64 \text{ lb} \)

and \( P = \frac{(9.64 \text{ lb})(30 \text{ ft/s})}{\left( \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb}} \right)} = 0.526 \text{ hp} \)

Into a 20 mph = 29.3 \( \frac{\text{ft}}{\text{s}} \) headwind \( P = \delta U_r \), where \( U_r = 30 \frac{\text{ft}}{\text{s}} = \text{runner's speed} \)

and \( \delta = C_d \frac{1}{2} \rho \left( U_r + 29.3 \frac{\text{ft}}{\text{s}} \right)^2 A \) or

\[
\delta = \left( \frac{1}{2} \right) (0.00238 \frac{\text{slugs}}{\text{ft}^2}) (30 + 29.3)^{\frac{2}{2}} (9 \text{ ft}^2) = 37.7 \text{ lb}
\]

Thus, \( P = \frac{(37.7 \text{ lb})(30 \text{ ft/s})}{\left( \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb}} \right)} = 2.06 \text{ hp} \)

With a 20 mph = 29.3 \( \frac{\text{ft}}{\text{s}} \) tailwind the relative headwind that the runner feels is \( U_r - 29.3 \frac{\text{ft}}{\text{s}} = (30 - 29.3) \frac{\text{ft}}{\text{s}} = 0.7 \frac{\text{ft}}{\text{s}} \)

Thus, \( \delta = \left( \frac{1}{2} \right) (0.00238 \frac{\text{slugs}}{\text{ft}^2}) (0.7 \frac{\text{ft}}{\text{s}})^2 (9 \text{ ft}^2) = 0.00525 \text{ lb} \)

Thus, \( P = \frac{(0.00525 \text{ lb})(30 \text{ ft/s})}{\left( \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb}} \right)} = 0.000286 \text{ hp} \)

Note: The tailwind essentially cancels the relative wind speed produced by the runner's forward motion.
9.83 A fishnet consists of 0.10-in.-diameter strings tied into squares 4 in. per side. Estimate the force needed to tow a 15 ft by 30 ft section of this net through seawater at 5 ft/s.

The net can be treated as one long 0.10-in.-diameter circular cylinder with

\[ \beta = C_d \frac{1}{2} \rho U^2 A \], where \( U = \frac{5 \text{ ft/s}}{15 \text{ ft}} \). Each 1 ft² section of the net contains 6 feet of string (don’t count the edges twice). Thus, the total string length is approximately \( L = (6 \frac{\text{ft}}{15 \text{ ft}})(15 \text{ ft})(30 \text{ ft}) = 2700 \text{ ft} \)

Also, since \( \rho = 1.99 \frac{\text{slug}}{\text{ft}^3} \) and \( V = 1.26 \times 10^{-5} \frac{\text{ft}^3}{\text{s}} \) (see Table 1.5)

\[ Re = \frac{U D}{V} = \frac{(5 \frac{\text{ft}}{15 \text{ ft}})(0.10 \frac{\text{in}}{\text{ft}})}{1.26 \times 10^{-5} \frac{\text{ft}^3}{\text{s}}} = 3310 \]

Hence, from Fig. 9.21 that \( C_d = 0.1 \)

Thus,

\[ \beta = (0.1)(\frac{1}{2})(1.99 \frac{\text{slug}}{\text{ft}^3})(5 \frac{\text{ft}}{15 \text{ ft}})^2 (0.1 \frac{\text{in}}{\text{ft}})(2700 \text{ ft}) = 616 \text{ lb} \]
As indicated in Fig. P9.84, the orientation of leaves on a tree is a function of the wind speed, with the tree becoming "more streamlined" as the wind increases. The resulting drag coefficient for the tree (based on the frontal area of the tree, $A$) as a function of Reynolds number (based on the leaf length, $L$) is approximated as shown. Consider a tree with leaves of length $L = 0.3$ ft. What wind speed will produce a drag on the tree that is 6 times greater than the drag on the tree in a 15 ft/s wind?

\[
\mathcal{D} = C_D \frac{1}{2} \rho U^2 A \quad \text{and} \quad \text{Re} = \frac{\rho UL}{\mu}
\]

or

\[
\mathcal{D} = C_D \left(0.00238\right) U^2 HW = 0.00119 HW C_D U^2
\]

and

\[
\text{Re} = \frac{0.0238 \frac{\text{slug}}{\text{lb.s/ft}^2} U (0.3 \text{H})}{3.74 \times 10^{-7} \text{lb.s/ft}^2} = 1909 \ U, \ \text{where} \ U \sim \text{ft/s}
\]

Thus, with $U = 15 \text{ ft/s}$, $\text{Re} = 1909 (15) = 28,600$ so that from Fig. P9.84,

$C_D = 0.46$ so

\[
\mathcal{D}_{15} = 0.00119 HW (0.46) (15)^2 = 0.123 HW
\]

For the drag 6 times as great,

\[
\mathcal{D} = 6 \mathcal{D}_{15} = 6 (0.123 HW) = 0.738 HW
\]

Thus, from Eqs. (1) and (3):

\[
0.738 HW = 0.00119 HW C_D U^2
\]

or

\[
C_D U^2 = 62.1
\]

**Trial and error solution:**

Assume $C_D = 0.3$ so that from Eq. (4), $U = \sqrt{\frac{62.1}{0.3}} = 45.5 \text{ ft/s}$ and from Eq. (2)

\[
\text{Re} = 1909 (45.5) = 86,900. \ \text{Thus, from Fig. P9.84,} \ C_D = 0.33 \pm 0.3, \ \text{the assumed value.}
\]

Try again. Assume $C_D = 0.33 \rightarrow U = 43.4 \text{ ft/s} \rightarrow \text{Re} = 82,900 \rightarrow C_D = 0.36 \pm 0.33$

Try $C_D = 0.36 \rightarrow U = 41.5 \text{ ft/s} \rightarrow \text{Re} = 79,300 \rightarrow C_D = 0.36$

Thus,

\[
U = 41.5 \text{ ft/s}
\]
A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft². Determine the lift coefficient of this airplane for these conditions.

For equilibrium \( \mathcal{L} = W = 1750 \text{ lb} \), where \( \mathcal{L} = C_L \frac{1}{2} \rho U^2 A \)

Thus, with \( U = (115 \text{ mph}) \frac{(169 \text{ ft})}{(60 \text{ mph})} = 169 \frac{\text{ft}}{s} \)

\[
C_L = \frac{\mathcal{L}}{\frac{1}{2} \rho U^2 A} = \frac{1750 \text{ lb}}{\frac{1}{2} (0.00238 \frac{\text{slugs}}{\text{ft}^3})(169 \frac{\text{ft}}{s})^2 (179 \text{ ft}^2)} = 0.288
\]

A light aircraft with a wing area of 200 ft² and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium \( \mathcal{L} = W = 2000 \text{ lb} = C_L \frac{1}{2} \rho U^2 A \)

or \( 2000 \text{ lb} = (0.40) \frac{1}{2} (0.00238 \frac{\text{slugs}}{\text{ft}^3}) U^2 (200 \text{ ft}^2) \)

Hence, \( U = 145 \frac{\text{ft}}{s} \)

Also, \( \mathcal{P} = \text{power} = \mathcal{D} U \), where

\[
\mathcal{D} = C_D \frac{1}{2} \rho U^2 A = (0.05) \frac{1}{2} (0.00238 \frac{\text{slugs}}{\text{ft}^3})(145 \frac{\text{ft}}{s})^2 (200 \text{ ft}^2) = 2.50 \text{ lb}
\]

Note: This value of \( \mathcal{D} \) could be obtained from

\[
\frac{W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8 \text{, or } \mathcal{D} = \frac{W}{8} = \frac{2000 \text{ lb}}{8} = 250 \text{ lb}
\]

Thus, \( \mathcal{D} = 250 \text{ lb} (145 \frac{\text{ft}}{s}) = 3.63 \times 10^4 \frac{\text{ft} \cdot \text{lb}}{s} \left( \frac{1 \text{ hp}}{550 \frac{\text{ft} \cdot \text{lb}}{s}} \right) = 65.9 \text{ hp} \)
9.87 As shown in Video V9.9 and Fig. P9.87, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is \( C_L = 1.1 \), and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.

Tractive force \( F_2 = \mu N_2 \)
where \( \mu = \text{coefficient of friction} = 0.6 \)

Thus, \( \Delta F_2 = \mu \Delta N_2 = \mu \Delta L \), where \( \Delta F_2 \) is the increase in tractive force due to the (downward) lift.

Hence, with \( V = 200 \text{ mph} = 293 \text{ ft/s} \),
\[ \Delta L = \frac{1}{2} \rho V^2 C_L A = \frac{1}{2} \left( 0.00238 \frac{\text{slugs}}{\text{ft}^3} \right) \left( 293 \text{ ft/s} \right)^2 \left( 1.1 \right) \left( 1.5 \text{ ft} \right) \left( 4 \text{ ft} \right) = 674 \text{ lb} \]
and
\[ \Delta F_2 = 0.6 \left( 674 \text{ lb} \right) = 405 \text{ lb} \]
The wings of old airplanes are often strengthened by the use of wires that provided cross-bracing as shown in Fig. P9.88. If the drag coefficient for the wings was 0.020 (based on the planform area), determine the ratio of the drag from the wire bracing to that from the wings.

\[ \frac{D_{\text{wire}}}{D_{\text{wing}}} = \frac{1}{2} \rho u^2 C_{d_{\text{wire}}} A_{\text{wire}} \]

and

\[ \frac{D_{\text{wire}}}{D_{\text{wing}}} = \frac{C_{d_{\text{wire}}} A_{\text{wire}}}{C_{d_{\text{wing}}} A_{\text{wing}}} \quad \text{so that} \]

\[ \frac{D_{\text{wire}}}{D_{\text{wing}}} = \frac{C_{d_{\text{wire}}} A_{\text{wire}}}{C_{d_{\text{wing}}} A_{\text{wing}}} \quad \text{where} \quad A_{\text{wing}} = 148 \text{ft}^2, \quad C_{d_{\text{wing}}} = 0.02 \]

Also, \[ A_{\text{wire}} = \ell \Delta = (160 \text{ ft})(\frac{0.05}{12 \text{ ft}}) = 0.667 \text{ ft}^2 \]

and since

\[ Re = \frac{UD}{\nu} = \left( \frac{70 \text{ mph}}{1.57 \times 10^{-6} \text{ ft}^2/\text{s}} \right) = 2720. \]

From Fig. 9.21, with \( Re = 2720 \) we obtain \( C_d = 1.0 \)

Hence,

\[ \frac{D_{\text{wire}}}{D_{\text{wing}}} = \frac{(1.0)(0.667\text{ft}^2)}{(0.02)(148\text{ ft}^2)} = 0.225 \quad \text{or} \quad 22.5\% \]
The jet engines on a Boeing 757 must develop a certain amount of power to propel the airplane through the air with a speed of 570 mph at a cruising altitude of 35,000 ft. By what percent must the power be increased if the same airplane were to maintain its 570 mph flight speed at sea level?

\[ P = \text{power} = \frac{1}{2} \rho U^3 C_{D_0} A \]

Let \( P_0 \) and \( \rho_{35} \) denote conditions at sea level and 35,000 ft, respectively. Thus, \( U_0 = U_{35} \) so that

\[ \frac{P_0}{\rho_{35}} = \frac{\frac{1}{2} \rho_0 U_0^3 C_{D_0} A_0}{\frac{1}{2} \rho_{35} U_{35}^3 C_{D_{35}} A_{35}} \]

so if \( A_0 = A_{35} \) and \( C_{D_0} = C_{D_{35}} \), then

\[ \frac{P_0}{\rho_{35}} = \frac{\rho_0}{\rho_{35}} \]

or with \( \rho \) values from Table C.1,

\[ \frac{P_0}{\rho_{35}} = \frac{0.00238 \text{ slugs/s}^2 \text{ ft}^2}{0.000738 \text{ slugs/s}^2 \text{ ft}^2} = 3.22 = 322\% \text{ increase} \]
A wing generates a lift \( L \) when moving through sea-level air with a velocity \( U \). How fast must the wing move through the air at an altitude of 35,000 ft with the same lift coefficient if it is to generate the same lift?

\[
L = C_L \frac{1}{2} \rho U^2 A \quad \text{so with} \quad L, C_L, \text{and} \ A \text{ constant}
\]

\[
(\rho U^2)_{\text{sea level}} = (\rho U^2)_{35,000 \text{ ft}}
\]

**Hence,**

\[
U_{35,000 \text{ ft}} = \left( \frac{\rho_{\text{sea level}}}{\rho_{35,000 \text{ ft}}} \right)^{\frac{1}{2}} U_{\text{sea level}} = \left( \frac{2.38 \times 10^{-3} \text{ slug/ft}^3}{7.38 \times 10^4 \text{ slug/ft}^3} \right)^{\frac{1}{2}} U_{\text{sea level}}
\]

or

\[
U_{35,000 \text{ ft}} = 1.80 U_{\text{sea level}}
\]
Air blows over the flat-bottomed, two-dimensional object shown in Fig. P9.91. The shape of the object, $y = y(x)$, and the fluid speed along the surface, $u = u(x)$, are given in the table. Determine the lift coefficient for this object.

If viscous effects are negligible, then

$$ \mathcal{L} = \int_0^L \rho \cos \Theta \, da - \int_{L_0}^{L_1} \rho \cos \Theta \, da $$

(1)

where from the Bernoulli equation

$$ \rho \frac{d^2 u}{dx^2} = \rho_0 + \frac{1}{2} \rho U^2 $$

The effect of atmospheric pressure, $\rho_0$, drops out when the integration over the entire surface is performed. With $\Theta = 0$ on the lower surface and with $\cos \Theta \, da = \cos \Theta \, (ds \, \ell) = \ell \, dx$, where $\ell = $ wing span, Eqs. (1) and (2) give

$$ \mathcal{L} = \int_0^L \rho \ell (u^2 - u_0^2) \cos \Theta \, dx $$

(2)

or, since $u = U$ on the lower surface

$$ \mathcal{L} = -\frac{1}{2} \rho \ell \int_{x = 0}^{x = 1} (u^2 - u_0^2) \, dx = \frac{L}{2} \rho U^2 \ell C \left( \frac{y}{b} \right)^2 - 1 \right) \, dx' $$

(3)

Thus, since

$$ C_L = \frac{\mathcal{L}}{\frac{1}{2} \rho U^2 \ell b} = \frac{\mathcal{L}}{\frac{1}{2} \rho U^2 \ell C} $$

it follows from Eq. (2) that

$$ C_L = \int_{x' = 0}^{x' = 1} \left( \frac{y}{b} \right)^2 - 1 \right) \, dx' $$

By using a standard numerical integration routine with the data given we obtain

$$ C_L = 0.327 $$

---

<table>
<thead>
<tr>
<th>$x' (% c)$</th>
<th>$y' (% c)$</th>
<th>$u/U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>3.72</td>
<td>0.971</td>
</tr>
<tr>
<td>5.0</td>
<td>5.30</td>
<td>1.232</td>
</tr>
<tr>
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<td>6.48</td>
<td>1.273</td>
</tr>
<tr>
<td>10</td>
<td>7.43</td>
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<td>11.14</td>
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<td>70</td>
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<td>1.26</td>
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<tr>
<td>100</td>
<td>0</td>
<td>0.807</td>
</tr>
</tbody>
</table>

---

$u \rightarrow$ upper surface

---

FIGURE P9.91
9.93 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight \( \mathcal{L} = C_L \frac{1}{2} \rho U^2 A = W \) \( \text{(1)} \)

Let \( \mathcal{L} \)\textsubscript{100} denote conditions with 100 passengers and \( \mathcal{L} \)\textsubscript{372} with 372 passengers. Thus, with \( C_L \)\textsubscript{100} = \( C_L \)\textsubscript{372}, \( A \)\textsubscript{100} = \( A \)\textsubscript{372}, and \( \rho \)\textsubscript{100} = \( \rho \)\textsubscript{372} Eq. (1) gives

\[
\frac{\mathcal{L} \textsubscript{100}}{\mathcal{L} \textsubscript{372}} = \frac{U \textsubscript{100}^2}{U \textsubscript{372}^2} \quad \text{or} \quad U \textsubscript{372} = U \textsubscript{100} \left( \frac{1}{580,000 + (372-100)(200)} \right)^{1/2}, \text{with} \ U \textsubscript{100} = 140 \text{mph}
\]
Thus, \( U \textsubscript{372} = 146 \text{ mph} \)

9.94 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, \( \theta \), is given by \( \tan \theta = C_D / C_L \).

For steady unpowered flight \( \Sigma F_x = 0 \) gives \( D = W \sin \theta \)
and \( \Sigma F_y = 0 \) gives \( \mathcal{L} = W \cos \theta \)

Thus,

\[
\frac{D}{\mathcal{L}} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta \quad \text{where} \quad \frac{D}{\mathcal{L}} = \frac{1}{2} \rho U^2 A C_D = \frac{C_D}{C_L}
\]

Hence, \( \tan \theta = \frac{C_D}{C_L} \)
9.95 If the lift coefficient for a Boeing 777 aircraft is 15 times greater than its drag coefficient, can it glide from an altitude of 30,000 ft to an airport 80 mi away if it loses power from its engines? Explain. (See Problem 9.94.)

\[
\text{From Problem 9.94, } \tan \theta = \frac{C_D}{C_L} = \frac{1}{15}
\]

Hence,
\[
\frac{30,000}{d} = \frac{1}{15}, \text{ or } d = 4.5 \times 10^5 \text{ ft} = 85.2 \text{ mi}
\]

Hence, the plane can glide 80 mi.

9.96 On its final approach to the airport an airplane flies on a flight path that is 3.0° relative to the horizontal. What lift-to-drag ratio is needed if the airplane is to land with its engines idled back to zero power? (See Problem 9.94.)

\[
\text{From Problem 9.94, } \tan \theta = \frac{C_D}{C_L}
\]

or
\[
\frac{C_D}{C_L} = \tan 3^\circ = 0.0524
\]

\[
\frac{C_L}{C_D} = 19.1
\]
9.99 The landing speed of an airplane such as the Space Shuttle is dependent on the air density. (See Video V9.1.) By what percent must the landing speed be increased on a day when the temperature is 110 deg F compared to a day when it is 50 deg F? Assume the atmospheric pressure remains constant.

For equilibrium, lift = weight, or

\[ \frac{1}{2} \rho U^2 C_L A = W \]

Thus, with constant \( W, C_L, \) and \( A, \)

\[ (\rho U^2)_{T=110} = (\rho U^2)_{T=50} \]

Or

\[ U_{110} = \left( \frac{\rho_{50}}{\rho_{110}} \right)^{\frac{1}{2}} U_{50} \]

But \( \rho = \rho RT \) so that

\[ \frac{\rho_{50}}{\rho_{110}} = \frac{(p_{50} / RT_{50})}{(p_{110} / RT_{110})} = \frac{(460 + 50)}{(460 + 110)} = 1.1176 \]

Thus,

\[ U_{110} = \sqrt{1.1176} U_{50} = 1.0572 \ U_{50} \text{ or a } 5.72\% \text{ increase} \]
9.97 A sail plane with a lift-to-drag ratio of 25 flies with a speed of 50 mph. It maintains or increases its altitude by flying in thermals, columns of vertically rising air produced by buoyancy effects of nonuniformly heated air. What vertical airspeed is needed if the sail plane is to maintain a constant altitude?

With no vertical air motion the sailplane would glide with a slope angle $\theta$, where since $\sum F = 0$

$\sum L = W \sin \theta$ and $\sum W = W \cos \theta$. Hence, $\frac{\sum L}{\sum W} = \tan \theta$

or since $\sum L = \frac{1}{2} \rho U^2 C_D A$ and $\sum W = \frac{1}{2} \rho U^2 C_L A$ it follows that $\tan \theta = \frac{C_D}{C_L}$. Therefore in still air the sailplane would lose altitude at a rate of $U \sin \theta$, where

$\theta = \tan^{-1} \left( \frac{C_D}{C_L} \right) = \tan^{-1} \left( \frac{1}{25} \right) = 2.29^\circ$. Hence an upward wind of (50 mph) $\sin 2.29^\circ$

= 2.00 mph will allow horizontal flight.
9.98 Over the years there has been a dramatic increase in the flight speed \((U)\) and altitude \((h)\), weight \((W)\), and wing loading \((W/A = \text{weight divided by wing area})\) of aircraft. Use the data given in the table below to determine the lift coefficient for each of the aircraft listed.

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Year</th>
<th>(W), lb</th>
<th>(U), mph</th>
<th>(W/A), lb/ft(^2)</th>
<th>(h), ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wright Flyer</td>
<td>1903</td>
<td>750</td>
<td>35</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>Douglas DC-3</td>
<td>1935</td>
<td>25,000</td>
<td>180</td>
<td>25.0</td>
<td>10,000</td>
</tr>
<tr>
<td>Douglas DC-6</td>
<td>1947</td>
<td>105,000</td>
<td>315</td>
<td>72.0</td>
<td>15,000</td>
</tr>
<tr>
<td>Boeing 747</td>
<td>1970</td>
<td>800,000</td>
<td>570</td>
<td>150.0</td>
<td>30,000</td>
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\[ C_L = \frac{2}{\rho U^2 A} = \frac{W}{\frac{1}{2} \rho U^2 A} = \frac{2}{\rho U^2} \left(\frac{W}{A}\right) \]

Thus:

<table>
<thead>
<tr>
<th></th>
<th>(\rho), slug/ft(^3)</th>
<th>(U), ft/s</th>
<th>(W/A), lb/ft(^2)</th>
<th>(C_L)</th>
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<tr>
<td>Wright Flyer</td>
<td>2.30 \times 10^{-3}</td>
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<td>1.76 \times 10^{-3}</td>
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<td>0.482</td>
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</table>
9.103 Boundary Layer on a Flat Plate

Objective: A boundary layer is formed on a flat plate when air blows past the plate. The thickness, \( \delta \), of the boundary layer increases with distance, \( x \), from the leading edge of the plate. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.103, to measure the boundary layer thickness.

Equipment: Wind tunnel; flat plate; boundary layer mouse consisting of ten Pitot tubes positioned at various heights, \( y \), above the flat plate; inclined multiple manometer; measuring calipers; barometer, thermometer.

Experimental Procedure: Position the tips of the Pitot tubes of the boundary layer mouse a known distance, \( x \), downstream from the leading edge of the plate. Use calipers to determine the distance, \( y \), between each Pitot tube and the plate. Fasten the tubing from each Pitot tube to the inclined multiple manometer and determine the angle of inclination, \( \theta \), of the manometer board. Adjust the wind tunnel speed, \( U \), to the desired value and record the manometer readings, \( L \). Move the boundary layer mouse to a new distance, \( x \), downstream from the leading edge of the plate and repeat the measurements. Record the barometer reading, \( H_{\text{bar}} \), in inches of mercury and the air temperature, \( T \), so that the air density can be calculated by use of the perfect gas law.

Calculations: For each distance, \( x \), from the leading edge, use the manometer data to determine the air speed, \( u \), as a function of distance, \( y \), above the plate (see Eq. 3.13). That is, obtain \( u = u(y) \) at various \( x \) locations. Note that both the wind tunnel test section and the open end of the manometer tubes are at atmospheric pressure.

Graph: Plot speed, \( u \), as ordinates and distance from the plate, \( y \), as abscissas for each location, \( x \), tested.

Results: Use the \( u = u(y) \) results to determine the approximate boundary layer thickness as a function of distance, \( \delta = \delta(x) \). Plot a graph of boundary layer thickness as a function of distance from the leading edge. Note that the air flow within the wind tunnel is quite turbulent so that the measured boundary layer thickness is not expected to match the theoretical laminar boundary layer thickness given by the Blasius solution (see Eq. 9.15).

Data: To proceed, print this page for reference when you work the problem and click here to bring up an EXCEL page with the data for this problem.

![Figure P9.103](con't)

9-92
9.100 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight $W = \text{aircraft weight} = \frac{L}{\rho} = C_\lambda \frac{1}{2} \rho U^2 A$

Thus, for given $W$, $C_\lambda$, and $A$ the dynamic pressure is constant, independent of altitude. That is

$\frac{\frac{1}{2} \rho U^2}{10,000 \text{ ft}} = \frac{\frac{1}{2} \rho U^2}{20,000 \text{ ft}}$, or $U_{20,000} = \left(\frac{20,000}{10,000}\right)^{\frac{1}{2}} U_{10,000}$

Hence, $U_{20,000} > U_{10,000}$

Also, since the drag is $\frac{D}{\rho} = C_D \frac{1}{2} \rho U^2 A$ it follows that

$\frac{D_{20,000}}{\rho U^2 A} = \frac{D_{10,000}}{\rho U^2 A}$, since $\frac{1}{2} U_{20,000} = \frac{1}{2} U_{10,000}$

Hence, the aircraft can fly faster at high altitudes with the same amount of drag ($D_{20,000} = D_{10,000}$)
9.102 For many years, hitters have claimed that some baseball pitchers have the ability to actually throw a rising fastball. Assuming that a top major leaguer pitcher can throw a 95-mpv pitch and impart a 1800-rpm spin to the ball, is it possible for the ball to actually rise? Assume the baseball diameter is 2.9 in. and its weight is 5.25 oz.

If the lift produced on the spinning ball is greater than its weight the ball will rise.

\[ L = C_L \frac{1}{2} \rho U^2 A \]

where \( C_L \) is a function of \( \frac{\omega D}{2U} \) as shown in Fig. 9.39.

Thus, with \( \frac{\omega D}{2U} = \frac{(188 \text{ rad/s})(2.9 \text{ ft})}{2(139 \text{ ft/s})} = 0.163 \)

\( C_L = 0.04 \)

Hence, for the given conditions

\[ L = 0.04 \left( \frac{1}{2} \right)(0.00238 \text{ slug}) \left( \frac{139 \text{ ft/s}}{\text{s}} \right)^2 \times \frac{(2.9 \text{ ft})^2}{(2.5 \text{ ft})^2} = 0.0422 \text{ lb} \]

so that

\[ L = 0.0422 \text{ lb} < W = 0.328 \text{ lb} \]

The ball will not rise.

**Note:** The above result is based on smooth-sphere data. The results for a baseball (with its rough surface containing seams) will probably give a somewhat larger lift because for a given angular velocity it can "drag" more air along as it spins.
### Solution for Problem 9.103: Boundary Layer on a Flat Plate

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<tr>
<th>$\theta$, deg</th>
<th>$H_{\text{atm}}$, in. Hg</th>
<th>$T$, deg F</th>
<th>$\gamma_{\text{H}_2\text{O}}$, lb/ft$^3$</th>
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<th>$u$, ft/s</th>
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<td>0.270</td>
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#### Data for $x = 3.75$ in.

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\[ \rho u^2/2 = \gamma_{\text{H}_2\text{O}} \cdot L \sin \theta \]

where

\[ \rho = \rho_{\text{atm}}/RT \]

\[ P_{\text{atm}} = \gamma_{\text{H}_2\text{O}} \cdot H_{\text{atm}} = 84/\text{lb/ft}^3 \times (29.09/12 \text{ ft}) = 2053 \text{ lb/ft}^3 \]

\[ R = 1716 \text{ ft lb/slug deg R} \]

\[ T = 80 + 460 = 540 \text{ deg R} \]

Thus, \( \rho = 0.00222 \) slug/ft$^3$

Approximate boundary layer thickness as obtained from the graph:

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(Con't)
Problem 9.103
Velocity, \( u \), vs Distance, \( y \)

Problem 9.103
Boundary Layer thickness, \( \delta \), vs Distance from Leading Edge, \( x \)

\[ \delta = 0.112x^{0.48} \]
9.104 Pressure Distribution on a Circular Cylinder

Objective: Viscous effect within the boundary layer on a circular cylinder cause boundary layer separation, thereby causing the pressure distribution on the rear half of the cylinder to be different than that on the front half. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.104, to determine the pressure distribution on a circular cylinder.

Equipment: Wind tunnel; circular cylinder with 18 static pressure taps arranged equally from the front to the back of the cylinder; inclined multiple manometer; barometer; thermometer.

Experimental Procedure: Mount the circular cylinder in the wind tunnel so that a static pressure tap points directly upstream. Measure the angle, $\beta$, of the inclined manometer. Adjust the wind tunnel fan speed to give the desired free stream speed, $U$, in the test section. Attach the tubes from the static pressure taps to the multiple manometer and record the manometer readings, $L$, as a function of angular position, $\theta$. Record the barometer reading, $H_{bar}$, in inches of mercury and the air temperature, $T$, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the data to determine the pressure coefficient, $C_p = (p - p_0)/(\rho U^2/2)$, as a function of position, $\theta$. Here $p_0 = 0$ is the static pressure upstream of the cylinder in the free stream of the wind tunnel, and $p = \gamma_{air} \sin \beta$ is the pressure on the surface of the cylinder.

Graph: Plot the pressure coefficient, $C_p$, as ordinates and the angular location, $\theta$, as abscissas.

Results: On the same graph, plot the theoretical pressure coefficient, $C_p = 1 - 4 \sin^2 \theta$, obtained from ideal (inviscid) theory (see Section 6.6.3).

Data: To proceed, print this page for reference when you work the problem and click here to bring up an EXCEL page with the data for this problem.
### Solution for Problem 9.104: Pressure Distribution on a Circular Cylinder

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<th>U, ft/s</th>
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<th>Theory</th>
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\[ p = \gamma_{\text{air}}^\text{L} \sin \beta \]

\[ p = p_{\text{atm}}/RT \text{ where} \]

\[ p_{\text{atm}} = \gamma_{\text{air}}^\text{L}H_{\text{atm}} = 847 \text{ lb/ft}^3(29.97/12 \text{ ft}) = 2115 \text{ lb/ft}^2 \]

\[ R = 1716 \text{ ft lb/slug deg R} \]

\[ T = 75 + 460 = 535 \text{ deg R} \]

Thus, \( p = 0.00230 \text{ slug/ft}^3 \)

\[ C_p = p/(\rho U^2/2) \]

Theory: \( C_p = 1 - 4 \sin^2 \theta \)
Problem 9.104
Pressure Coefficient, $C_p$, vs Angle, $\theta$

- Experimental
- Theoretical (inviscid flow)
9.105 (See "Armstrong's aerodynamic bike and suit," Section 9.1.) By appropriate streamlining, the amount of power needed to peddle a bike can be lowered. How much must the drag coefficient for a bike and rider be reduced if the power a bike racer expends while riding 13 m/s is to be reduced by 10 watts? Assume the cross-sectional area of the bike and rider is 0.36 m².

\[ \mathcal{D} = C_d \frac{1}{2} \rho U^2 A \quad \text{and} \quad P = \text{power} = \mathcal{D} U \]

Thus,

\[ \Delta P = \frac{1}{2} \rho U^3 A \Delta C_d \]

or

10 watts = 10 N·m/s = \( \frac{1}{2} \times (1.23 \text{ kg/m}^3) \times (13 \text{ m/s})^3 \times (0.36 \text{ m}^2) \Delta C_d \)

Hence,

\[ \Delta C_d = 0.0206 \]

Note from Fig. 9.30 the value of the drag coefficient for a racing bike is about 0.88.
9.108 (See "Learning from nature," Section 9.4.1.) As indicated in Fig. P9.108, birds can significantly alter their body shape and increase their planform area, \( A \), by spreading their wing and tail feathers, thereby reducing their flight speed. If during landing the planform area is increased by 50% and the lift coefficient increased by 30% while all other parameters are held constant, by what percent is the flight speed reduced?

\[
\mathcal{L} = C_L \frac{1}{2} \rho U^2 A
\]

Let \((\cdot)_2\) denote landing conditions and \((\cdot)_1\) denote normal flight conditions. Thus, with \( \mathcal{L}_1 = \mathcal{L}_2 \),

\[
C_L \frac{1}{2} \rho U^2 A_2 = C_L \frac{1}{2} \rho U_2^2 A_2
\]

or

\[
U_2 = U_1 \sqrt{\frac{A_1}{A_2}} = U_1 \sqrt{\frac{\mathcal{L}_1}{1.5 \mathcal{L}_2}} = \sqrt{\frac{A_1}{1.3 A_2}}
\]

or

\[
U_2 = 0.716 \ U_1
\]

Hence,

\[
\frac{U_2 - U_1}{U_1} = 0.716 - 1 = -0.284
\]

i.e., a 28.4% reduction in flight speed
9.109 (See "Why winglets?" Section 9.4.2.) It is estimated that by installing appropriately designed winglets on a certain airplane the drag coefficient will be reduced by 5%. For the same engine thrust, by what percent will the aircraft speed be increased by use of the winglets?

Let \( (\cdot)_1 \) denote without winglets and \( (\cdot)_2 \) with winglets. Thus, since drag equals thrust and \( \text{thrust}_1 = \text{thrust}_2 \), it follows that
\[
\frac{\delta}{C} = \frac{\delta'}{C'}
\]
or
\[
\frac{C}{\rho U^2 A_1} = \frac{C'}{\rho U^2 A_2}
\]
so that with \( A_1 = A_2 \),
\[
U_2 = U_1 \sqrt{\frac{C}{\rho U^2 A_2}} = U_1 \sqrt{\frac{C}{0.95 C_0}} = 1.026 U_1
\]
Thus, a 2.60% increase in speed is realized.
9.106 (See “Dimpled baseball bats,” Section 9.3.3.) How fast must a 3.5-in.-diameter, dimpled baseball bat move through the air in order to take advantage of drag reduction produced by the dimples on the bat. Although there are differences, assume the bat (a cylinder) acts the same as a golf ball in terms of how the dimples affect the transition from a laminar to a turbulent boundary layer.

From Fig. 9.25, for a golf ball the dimples reduce drag for \( \text{Re} = \frac{QUD}{\mu} \geq 4 \times 10^4 \)

Thus, assume \( \text{Re} = 4 \times 10^4 \) for the bat so that

\[
\frac{QUD}{\mu} = 4 \times 10^4
\]

or

\[
\left( \frac{0.00238 \text{slug/s}}{\text{ft}^3} \right) U \left( \frac{3.5 \text{ ft}}{\text{ft}} \right) = 4 \times 10^4
\]

Thus,

\[
U = 21.6 \text{ ft/s}
\]
9.107 (See "At 10,240 mpg it doesn’t cost much to ‘fill ‘er up’,” Section 9.3.3.) (a) Determine the power it takes to overcome aerodynamic drag on a small (6 ft$^2$ cross section), streamlined ($C_D = 0.12$) vehicle traveling 15 mph. (b) Compare the power calculated in part (a) with that for a large (36 ft$^2$ cross-sectional area), nonstreamlined ($C_D = 0.48$) SUV traveling 65 mph on the interstate.

\[ \mathcal{P} = \text{power} = U \delta, \quad \delta = C_D \frac{1}{2} \rho U^2 A \]

so that

\[ \mathcal{P} = C_D \frac{1}{2} \rho U^2 A \]

(a) \[ \mathcal{P} = 0.12 \left( \frac{1}{2} \right) (0.00238 \text{ slugs/ft$^3$}) \left( 15 \text{ mi/hr} \right) \left( \frac{5280 \text{ ft/mi}}{3800 \text{ s/hr}} \right)^3 (6 \text{ ft}$^2$) \]

\[ = 9.12 \frac{ft-lb}{s} \left( \frac{1 \text{ hp}}{550 \text{ ft-lb/s}} \right) = 0.0166 \text{ hp} \]

(b) \[ \mathcal{P} = 0.48 \left( \frac{1}{2} \right) (0.00238 \text{ slugs/ft$^3$}) \left( 65 \text{ mi/hr} \right) \left( \frac{5280 \text{ ft/mi}}{3600 \text{ s/hr}} \right)^3 (36 \text{ ft}$^2$) \]

\[ = 17,800 \frac{ft-lb}{s} \left( \frac{1 \text{ hp}}{550 \text{ ft-lb/s}} \right) = 32.4 \text{ hp} \]